Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

Seat No.	

[4457]-14

S.E. (Mechanical/Production/Industrial Automobile)

(First Semester) EXAMINATION, 2013

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^4 - 1)y = \cosh x \,.\, \sinh x$

(*ii*) $(D^2 + 5D + 4)y = x^2 + 7x + 9$

(*iii*) $(D^2 + 6D + 9)y = \frac{1}{x^3} e^{-3x}$.

(*iv*) $\frac{d^2y}{dx^2} + 9y = \text{cosec } 3x$

(By using variation of parameter method)

P.T.O.

(*b*) Solve the symmetric simultaneous differential equation : [5]

$$\frac{dx}{x^2 + y^2} = \frac{dy}{2xy} = \frac{dz}{(x+y)^3 z}.$$

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^3 + 1)y = \sin(x + 5)$

(*ii*) $(D^2 + 2D + 1)y = x \cos x$

(*iii*) $x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \sin(\log x)$

(*iv*) $(D^2 - 4D + 3)y = e^x \cos 2x.$

(*b*) Solve the system of simultaneous equations : [5]

$$\frac{dx}{dt} + y = e^t$$

$$\frac{dy}{dt} - x = e^{-t}.$$

3. (*a*) Find Laplace transform (any *two*) of the following : [6]

(*i*) $e^{-t} \sin^3 t$

(*ii*) $t\sqrt{1 + \sin t}$

(*iii*) $\frac{d}{dt}\left(\frac{\sin t}{t}\right).$

(*b*) Solve : [5]

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$$

where $y(0) = 3$, $y'(0) = 6$, using Laplace transform method.

(*c*) Solve the integral equation : [5]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0.$$

Or

4. (*a*) Find Inverse Laplace transform (any *two*) of the following : [8]

(*i*) $\tan^{-1}\frac{1}{s}$

(*ii*) $\frac{s+2}{s^2 (s+3)}$

(*iii*) $e^{-5s} \frac{1}{(s-2)^4}$.

(*b*) Evaluate $\int_0^\infty t \, e^{-2t} \cos t \, dt$ using Laplace transform. [4]

(*c*) Find Inverse Fourier sine transform if : [4]

$$F_s(\lambda) = \begin{cases} 2-\lambda, & 0 \le \lambda \le 2 \\ 0, & \lambda > 2 \end{cases}.$$

5. (a) Solve : [8]

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$$

subject to the conditions :

(i) $y(0, t) = 0$

(ii) $y(\pi, t) = 0$

(iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(iv) $y(x, 0) = 0.1 \sin x + 0.01 \sin 4x, \; 0 \le x \le \pi$

(b) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is : [9]

$$u(x, 0) = x, \qquad 0 \le x \le 50$$
$$= 100 - x \qquad 50 \le x \le 100$$

Find the temperature $u(x, t)$ at any time.

Or

6. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with conditions : [9]

(i) $u = 0$ when $y \to \infty$ for all x

(ii) $u = 0$ when $x = 0$ for all y

(iii) $u = 0$ when $x = 1$ for all y

(iv) $u = x(1 - x)$ when $y = 0$ for $0 < x < 1$.

(*b*) Use Fourier transform to solve : [8]

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

subject to

(*i*) $u_x = 0$ at $x = 0$ for all t

(*ii*) $u(x, 0) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x > 1 \end{cases}$.

SECTION II

7. (*a*) Lives of two models of refrigerators turned for new models in a recent year are : [6]

Life	**No. of Refrigerators**	
No. of Years	**Model A**	**Model B**
0—2	5	2
2—4	16	7
4—6	13	12
6—8	7	9
8—10	5	9
10—12	4	1

Find which model has more uniformity ?

(*b*) Find first moments about the mean of the following : [6]

x	f
61	5
64	18
67	42
70	27
73	8

Also find mean and standard deviation.

(*c*) Mean and variance of Binomial distribution are 4 and 2 respectively. Find $P(r \geq 2)$. [4]

Or

8. (*a*) If two lines of regression are [5]

$$9x + y - \lambda = 0 \text{ and } 4x + y = \mu$$

and means of x and y are 2 and -3 respectively.

Find :

(*i*) λ and μ

(*ii*) coefficient of correlation between x and y.

(*b*) Prove that the following data represent Poisson distribution : [5]

x	f
0	109
1	65
2	22
3	3
4	1

(*c*) Among 64 offsprings of a certain cross between European horses, the following observations are made : [6]

Red	Black	White
34	10	20

According to genetic model, these numbers should be in the ratio 9 : 3 : 4.

Is the data consistent with the model at 5% level ?

Given : $\chi^2_{2, 0.05} = 5.991$.

9. (*a*) Find the angle between the tangents to the curve : [5]

$$\bar{r} = (t^3 + 2)i + (4t - 5)j + (2t^2 - 6t)k$$

at $t = 0$ and $t = 2$.

(*b*) Find directional derivative of [6]

$$\phi = 4xz^3 - 3x^2y^2z$$

at (2, –1, 2) along the tangent to the curve

$$x = e^t \cos t,\ y = e^t \sin t,\ z = e^t \quad \text{at} \quad t = 0.$$

(*c*) If [6]

$$\overline{\mathrm{F}} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$$

is irrotational, find a, b, c and determine ϕ such that $\overline{\mathrm{F}} = \nabla\phi$.

Or

10. (*a*) Show that the following (any *two*) : [8]

(*i*) $\nabla^2\left[\nabla \cdot \dfrac{\bar{r}}{r^2}\right] = \dfrac{2}{r^4}$

(*ii*) $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$

(*iii*) $\nabla \cdot \left(\dfrac{\bar{a} \times \bar{r}}{r}\right) = 0.$

(*b*) If $\bar{r} = \bar{a} \sinh t + \bar{b} \cosh t$, where $\bar{a}, \bar{b}$ are constant, then prove that : [4]

(*i*) $\dfrac{d^2\bar{r}}{dt^2} = \bar{r}$

(*ii*) $\dfrac{d\bar{r}}{dt} \times \dfrac{d^2\bar{r}}{dt^2} = \bar{a} \times \bar{b}.$

(*c*) If [5]

$$\bar{E} = (y + z)i + (z + x)j + (x + y)k$$

show that :

$$\text{curl curl curl curl } \bar{E} = \nabla^4 \bar{E}.$$

11. (*a*) If [6]

$$\bar{F} = 3x^2 i + (2xz - y)j + zk$$

evaluate :

$$\int_C \bar{F} \cdot d\bar{r}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining the points (0, 0, 0) and (1, 1, 1).

(*b*) Verify Gauss-divergence theorem for the closed surface S bounded by [6]

$$x^2 + y^2 = 4, z = 0, z = 2$$

where :

$$\bar{F} = xi + yj + z^2 k.$$

(*c*) Evaluate : [5]

$$\iint_S \nabla \times \bar{F} \cdot d\bar{S}$$

for $\bar{F} = yi + zj + xk$, where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$.

Or

12. (*a*) Verify Stokes' theorem for [7]

$$\bar{F} = (2x - y)i - yz^2 j - y^2 zk$$

where S is upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.

(*b*) Evaluate [5]

$$\int_C \bar{F} . d\bar{r}$$

where

$$\bar{F} = \sin yi + x (1 + \cos y) j$$

and

C is $x^2 + y^2 = 1, z = 0.$

(*c*) Evaluate [5]

$$\iint_S \bar{r} . \hat{n} \, ds$$

over the surface of a sphere of radius 1 with centre at origin.