

Total No. of Questions : 8]

SEAT No. :

P2680

[Total No. of Pages : 3

M.E. (Mechanical) (CADME)
MATHEMATICAL MODELLING
(2008 Pattern) (Semester-I) (202050)

Time : 3 Hours]

[Max. Marks : 100

Instructions to the candidates:

- 1) *Answers to the two sections should be written in separate answer books.*
- 2) *Answer any three questions from each section.*
- 3) *Neat diagrams must be drawn wherever necessary.*
- 4) *Figures to the right side indicate full marks.*
- 5) *Use of Calculator is allowed.*
- 6) *Assume Suitable data if necessary.*

SECTION-I

Q1) a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subjected to the condition

$u(0, y) = D, u(u, y) = 8 + 2y \quad Y(x, 0) = \frac{x^2}{2}, u(x, u) = x^2$ with $\Delta x = \Delta y = 1$ in
 $u < x < 4, u < y < 4.$ [9]

b) A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in a position given by $y = y_0 \sin^3 [\Pi x / l]$. If it is released from rest from this position, find displacement $y(x, t)$. [9]

Q2) a) Using given matrix reduce the following matrix to the tridiagonal form. [6]

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$$

b) Solve the boundary value problem by Ritz Method [10]

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 2x^2, \quad y(0) = 1, \quad y(1) = 1.$$

P.T.O.

Q3) a) Find the Extremals of the isoperimetric problem $I[y(x)] = \int_{x_0}^{x_1} y^2 dx$. Given

that $\int_{x_0}^{x_1} y dx = c$, a constant. [6]

b) Using Jacobis method to find eigen values of the matrix [10]

$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}$$

Q4) a) Find the deflection of the vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = K (\sin x - \sin 2x)$ [6]

b) Solve the boundary value problem using Galerkins Method $Y'' + Y = e^x$

where $Y'' = \frac{d^2 y}{dx^2}$. [10]

SECTION-II

Q5) The weight of the calf taken at weekly interval are given below. Fit a straight line using the method at least square and find the average rate of growth for a week. [18]

Age	1	2	3	4	5	6	7	8	9	10
Weight	52.5	58.7	65	70.2	75.4	81.1	87.2	96.5	102.2	108.4

Q6) a) Find the smallest eigen value by inverse iteration method [8]

$$\begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix} [x] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using starting vector $\{x\}^T = \langle 1111 \rangle$.

- b) The following table gives the values of $f(x)$ at the equal interval of x . [8]

x	0	0.5	1	1.5	2
$f(x)$	0.399	0.352	0.242	0.129	0.054

- Q7) a) Using Laczos method calculate two smallest eigen values and eigen vectors of [6]

$$\begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix} [x] = \lambda \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Using starting vector $\{x\}^T = \langle 11111 \rangle$.

- b) Determine the size of the simple for estimating the true weight of the cerial containers for the universe with $N = 5000$ on the basis of the following information [10]
- the variance of weight = 4 Dunces on the basis of past record
 - estimate should be within 0.8 Dunces of the true average weight withing 99% probability.

- Q8) a) Weight of the 10 students is as follows. [10]

Sr	1	2	3	4	5	6	7	8	9	10
Weight	18	40	45	53	47	43	55	48	52	49

Can we say that the variance of the distribution of weight of all students from which the above sample of 10 students was drawn is equal to 20kg? Test this at 5% and 1% level of significance.

- b) Write the characteristics of hypothesis. [6]

