

Total No of Questions: [12]

SEAT NO. :

[Total No. of Pages : 6]

B.E. Mechanical (2008 Pattern)

Finite Element Method

(Elective - III) (Semester - II)

Time: 3 Hours

Max. Marks : 100

Instructions to the candidates:

- 1) Answers to the two sections should be written in separate answer books.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right side indicate full marks.
- 4) Use of Calculator is allowed.
- 5) Assume Suitable data if necessary.
- 6) Additional data sheet is attached for the reference.

SECTION I

Unit I

Q1)	a)	Explain how banded skyline solution method is used to solve simultaneous equations.	[8]
	b)	List at least 6 advantages of Finite Element Method over analytical method. Also list disadvantages or limitations of FEM.	[8]

OR

Q2)	a)	Explain the terms essential and natural boundary conditions. Give example of each.	[8]
	b)	Explain in detail the method of matrix partitioning and how it is used to impose boundary conditions in finite element method.	[8]

Unit II

Q3)	a)	For the plane truss shown in Figure 3a, determine the following. Each element has $E = 70 \text{ GPa}$, and area $A = 200 \text{ mm}^2$. i. write down the elemental stiffness matrices (k) for each element, ii. assemble k matrices to get global stiffness matrix (K), iii. apply boundary conditions, iv. find horizontal and vertical displacements of node 1, v. determine reaction forces at node 2, 3 and 4.	[10]
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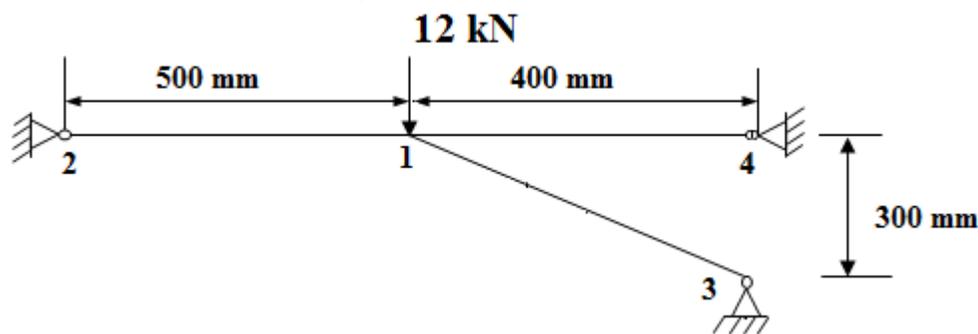


Figure 3a

	b)	For the five spring assemblage shown in Figure 3b, determine the displacements at nodes 2 and 3 and the reactions at nodes 1 and 4. Assume the rigid vertical bars at nodes 2 and 3 connecting the springs remain horizontal at all times but are free to slide or displace left or right. There is an applied force at node 3 of 1000 N to the right. Consider $k_1 = 500 \text{ N/mm}$, $k_2 = k_3 = 300 \text{ N/mm}$, and $k_4 = k_5 = 400 \text{ N/mm}$	[8]
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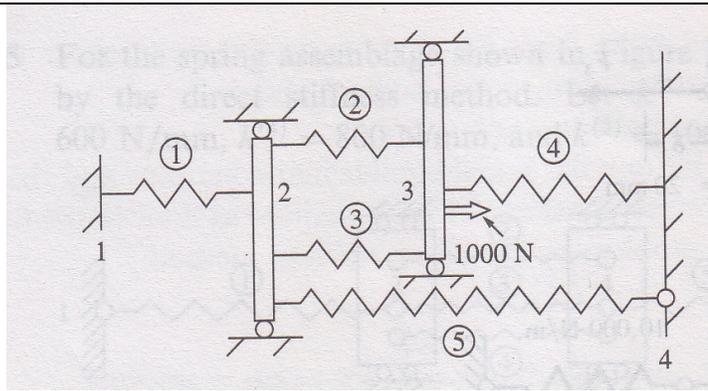


Figure 3b

OR

Q4) a) Derive elemental stiffness matrix and force vector for two-noded (linear) bar element using Principle of Minimum Potential Energy (PMPE) Method. The bar element is oriented in x-direction. Assume that only concentrated forces are acting on the nodes of bar element. Ignore surface tractions and the body forces. [10]

b) For the beam shown in Figure 4b, determine the displacements and the slope at node 2. Also find the reaction forces and moments at nodes 1 and 3. Consider $E = 210 \text{ GPa}$, and $I = 4 \times 10^{-4} \text{ m}^4$ [8]

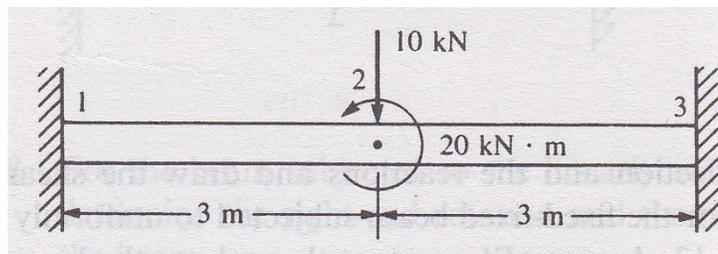


Figure 4b

Unit III

Q5) Evaluate the following integrals.

a) use two-point Gaussian quadrature method [5]

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

b) use two-point Gaussian quadrature method [5]

$$I = \int_1^5 [3^x + 1] dx$$

c) use three-point Gaussian quadrature method [6]

$$I = \int_4^5 \frac{2 \sin(x)}{x^2} dx$$

OR

Q6) a) Explain with neat sketches the difference between p and h refinements in Finite Element Method. [6]

b) For the three-noded iso-parametric bar element shown in Figure 6b, show that the Jacobean determinate is $|J| = L/2$. Also determine the shape functions $N1 - N3$ and the strain-displacement matrix $[B]$. Assume the displacement field as $u = a1 + a2s + a3s^2$.

[10]

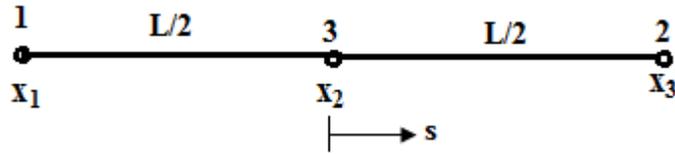


Figure 6b

SECTION II

Unit IV

Q7) Determine the temperature distribution along the length of the rod (at $L/4$, $L/2$, $3L/4$, and L) as shown in Figure 7. The rod with radius of 25 mm is insulated at the perimeter. The left end has a constant temperature of 40°C and a free stream temperature T_∞ is -10°C . Let $K_{xx} = 35 \text{ W}/(\text{m}^\circ\text{C})$ and $h = 55 \text{ W}/(\text{m}^2^\circ\text{C})$.

[16]

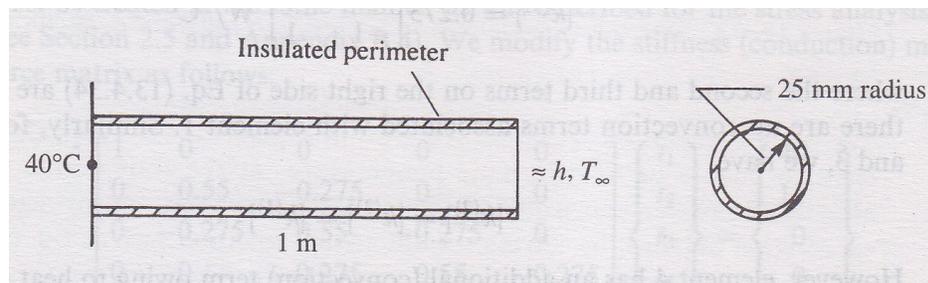


Figure 7

OR

Q8) A composite wall shown in Figure 8 is composed of two homogeneous slabs in contact. Let thermal conductivities be $K1 = 1 \text{ W}/(\text{m}^\circ\text{C})$ for firebrick slab 1 and $K2 = 0.3 \text{ W}/(\text{m}^\circ\text{C})$ for insulating slab 2. The left side is exposed to an ambient temperature of $T_{\infty L} = 1000^\circ\text{C}$ inside the furnace with heat transfer coefficient of $hL = 10 \text{ W}/(\text{m}^2^\circ\text{C})$. The right side ambient temperature is $T_{\infty R} = 25^\circ\text{C}$ outside of the furnace with heat transfer coefficient of $hR = 3 \text{ W}/(\text{m}^2^\circ\text{C})$. The thicknesses of the slabs are $L1 = 0.20 \text{ m}$, and $L2 = 0.10 \text{ m}$. Determine the temperature at the left edge, point between the two slabs and right edge of the composite wall.

[16]

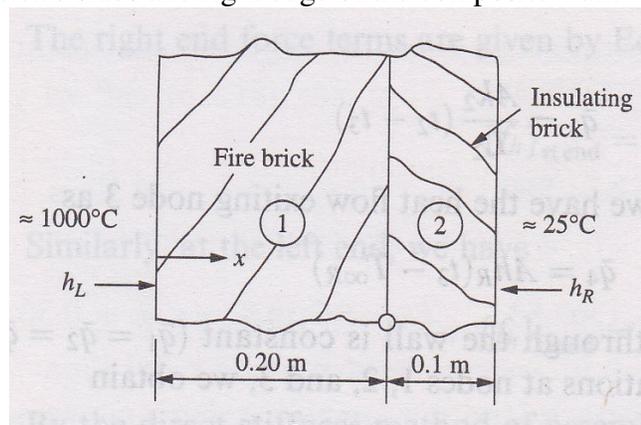
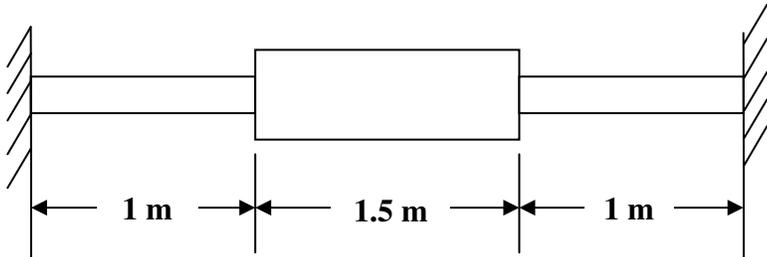


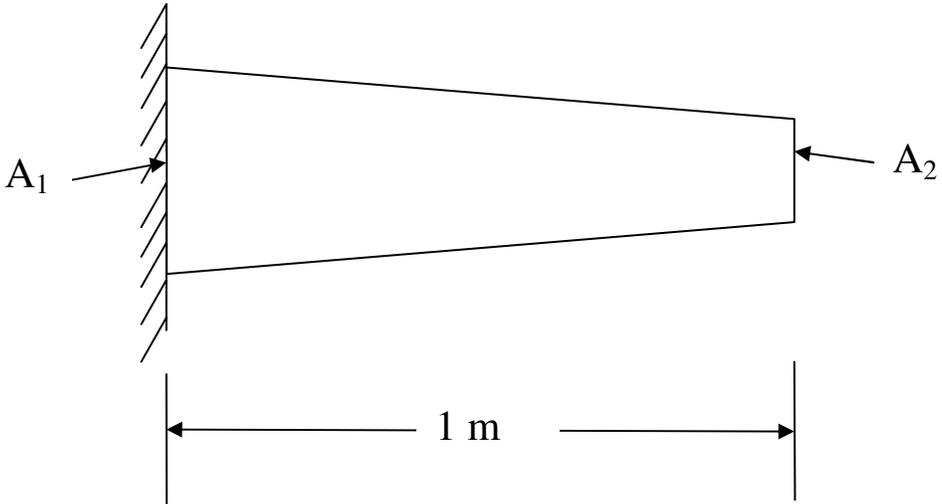
Figure 8

Unit V

Q9)	a)	<p>For the stepped bar shown in Figure 9a, determine the first two natural frequencies in terms of rad/s for un-damped free vibration. Let $A_1 = A_3 = 5 \text{ cm}^2$, $A_2 = 10 \text{ cm}^2$, $E = 210 \text{ GPa}$, and $\rho = 7860 \text{ kg/m}^3$. Use consistent mass matrices for each element.</p>  <p style="text-align: center;">Figure 9a</p>	[12]
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	b)	Explain eigenvalue problem for un-damped free vibration system	[6]
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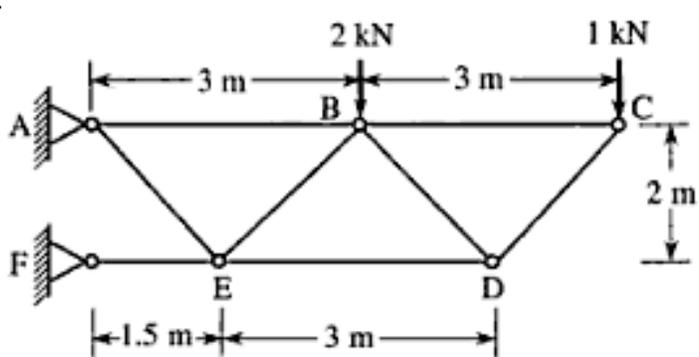
OR

Q10)	a)	<p>For the tapered beam with square cross sectional area shown in Figure 10, set-up the problem to find characteristic equation to determine first four natural frequencies. Consider two elements of average cross sectional area and equal lengths. Each element has $E = 200 \text{ GPa}$, density $\rho = 7500 \text{ kg/m}^3$, $A_1 = 9 \text{ cm}^2$, and $A_2 = 4 \text{ cm}^2$. Use consistent mass matrix.</p>  <p style="text-align: center;">Figure 10</p>	[18]
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Unit VI

Q11)	a)	What are various meshing techniques?	[8]
	b)	Explain the terms strain and stress recovery in post-processing stage.	[8]

OR

Q12)	<p>Consider the truss problem shown in Figure 12 for the calculation of displacements. For this problem write down nodal coordinates, element connectivity, type of analysis, loading and boundary conditions.</p>  <p style="text-align: center;">Figure 12</p>	[16]
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DATA SHEET

Shape Functions:

1 Bar Element:

$$N_1 = 1 - \frac{x}{L} \quad N_2 = \frac{x}{L}$$

2 Beam Element:

$$N_1 = \frac{1}{L^3}(2x^3 - 3x^2L + L^3)$$

$$N_2 = \frac{1}{L^3}(x^3L - 2x^2L^2 + xL^3)$$

$$N_3 = \frac{1}{L^3}(-2x^3 + 3x^2L)$$

$$N_4 = \frac{1}{L^3}(x^3L - x^2L^2)$$

Elemental Stiffness Matrices:

1 Bar Element:

$$k_{bar} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2 Beam Element:

$$k_{beam} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3 Truss Element:

$C = \cos(\theta)$ and $S = \sin(\theta)$
 θ is positive in anti clockwise direction.

$$k_{truss} = \frac{AE}{L} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

Elemental Mass Matrices:

1 Bar Element:

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

	(b) Lumped mass matrix:	
		$m_{lumped} = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2 Beam Element:

	(a) Consistent mass matrix:	
		$m_{consistent} = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$

	(b) Lumped mass matrix:	
		$m_{lumped} = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Heat Transfer Matrices:

k matrix for Conduction + Convection for bar element:

$$k = \frac{AK}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

where, A = cross sectional area, K = Thermal Conductivity, L = Length of an element, h = Convection Coefficient, and P = Perimeter.

Gauss Quadrature:

Table for Gauss Points for integration from -1 to 1

$$\int_{-1}^1 y(x)dx = \sum_{i=1}^n W_i y_i$$

Number of Points	Locations, x_i	Associated Weights, W_i
1	$x_1 = 0.000$	2.000
2	$x_1, x_2 = \pm 0.57735$	1.000
3	$x_1, x_3 = \pm 0.77459$ $x_2 = 0.000$	$5/9 = 0.55556$ $8/9 = 0.88889$
4	$x_1, x_4 = \pm 0.86113$ $x_2, x_3 = \pm 0.33998$	0.34785 0.65214