[4656] – 101

Seat No.	

F.E. (Semester – I) Examination, 2014
ENGG. MATHEMATICS – I
(2012 Pattern)

Time : 2 Hours   Max. Marks : 50

***Instructions** : 1) Attempt Q. **1** or Q. **2**, Q. **3** or Q. **4**, Q. **5** or Q. **6**, Q. **7** or Q. **8**.*
*2) Neat diagrams must be drawn **wherever** necessary.*
*3) Black figures to the **right** indicate full marks.*
*4) Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is **allowed**.*
*5) Assume suitable data, if **necessary**.*

1. A) Examine for consistency the system of equations

$$x - y - z = 2$$
$$x + 2y + z = 2$$
$$4x - 7y - 5z = 2$$

and solve it if consistent. **4**

B) Examine whether the following vectors are linearly dependent or independent. If dependent, find the relation between them.

$X_1 = [1\ \ -1\ \ 2]$ $X_2 = [2\ \ 3\ \ 5]$ $X_3 = [3\ \ 2\ \ 1]$ **4**

C) If cosec (x + iy) = u + iv , prove that **4**

i) $\dfrac{u^2}{\sin^2 x} - \dfrac{v^2}{\cos^2 x} = (u^2 + v^2)^2$

ii) $\dfrac{u^2}{\cosh^2 y} + \dfrac{v^2}{\sinh^2 y} = (u^2 + v^2)^2$

OR

2. A) A square lies above real axis in Argand diagram and two of its adjacent vertices are the origin and the point 2 + 3i. Find the complex numbers representing other two vertices. **4**

B) If arg $(z+1) = \dfrac{\pi}{6}$ and arg $(z-1) = \dfrac{2\pi}{3}$ then find z. **4**

C) Find the eigen values and eigen vectors of following matrix. **4**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

P.T.O.

3. a) Test convergence of the series (**any one**). **4**

   i) $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{28}}+\frac{1}{\sqrt{65}}+.....$

   ii) $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+.....$

   b) Prove that log (1 + sinx) = $x-\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{12}+....$ **4**

   c) Find n^{th} derivative of $\frac{x^2}{(x-1)(x-2)}$. **4**

OR

4. a) Solve **any one**. **4**

   i) Evaluate $\lim_{x\to o} \log_{\tan x} \tan 4x$.

   ii) Find the values of a and b if $\lim_{n\to o} \left[x^{-3}\sin x + ax^{-2} + b\right] = 0$.

   b) Using Taylor's theorem expand 49 + 69 x + 42 x^2 + 11 x^3 + x^4 in powers of (x+2). **4**

   c) If y = sin log (x^2 + 2 x + 1), then prove that

   $(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2+4) y_{n=0}$ **4**

5. Solve **any two** of the following.

   a) If $u=\log(x^3+y^3-x^2y-xy^2)$ then prove that x^2 u_{xx} + 2_{xy} u_{xy} + y^2 u_{yy} = – 3. **7**

   b) If x = u + v + w, y = uv + vw + wu, z = uvw and ϕ is a function of x, y, z then prove that

   $u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+w\frac{\partial\phi}{\partial w}=x\frac{\partial\phi}{\partial x}+2y\frac{\partial\phi}{\partial y}+3z\frac{\partial\phi}{\partial z}$. **6**

   c) If ux+vy = 0 and $\frac{u}{x}+\frac{v}{y}=1$, then prove that $\left(\frac{\partial u}{\partial x}\right)_y-\left(\frac{\partial v}{\partial y}\right)_x=\frac{x^2+y^2}{y^2-x^2}$. **6**

OR

6. Solve **any two** of the following.

   a) If $u=\cos\left(\frac{xy}{x^2+y^2}\right)+\sqrt{x^2+y^2}+\frac{xy^2}{x+y}$, then find the value of xu_x+ yu_y at (3 ,4). **7**

   b) If $x=\frac{\cos\theta}{u}$, $y=\frac{\sin\theta}{u}$, then prove that $u\frac{\partial z}{\partial u}-\frac{\partial z}{\partial\theta}=(y-x)\frac{\partial z}{\partial x}-(y-x)\frac{\partial z}{\partial y}$. **6**

   c) If $u=(x^2-y^2)\, f(xy)$, then show that $u_{xx} - u_{yy}$ = (x^4-y^4) $f''(xy)$. **6**

7. a) If x = r sinθ cosϕ, y = r sinθ sin ϕ, z = r cosθ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **4**

b) Examine for functional dependence $u = \sin^{-1}x + \sin^{-1}y,\ \ v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ if dependent find the relation between them. **4**

c) The area of a triangle ABC is calculated from the formula $\Delta = \frac{1}{2}bc\,\sin A$. Errors of 1%, 2% and 3% respectively are made in measuring b, c, A. If the correct value of A is 30°, find the percentage error in the calculated value of area of triangle. **5**

OR

8. a) If $u^2 + xv^2 - uxy = 0$, $v^2 - xy^2 + 2uv + u^2 = 0$, find $\frac{\partial u}{\partial x}$ by choosing u , v as dependent and x, y as independent variables. **4**

b) Show that $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ are functionally dependent and find the relation between them. **4**

c) Find all the stationary values of the function f (x, y) = $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. Find maximum value of f(x, y) at suitable point. **5**

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