

Total No of Questions: [08]

SEAT NO. :

[4456]-101

F.E. (2012)

Engg. Mathematics – I

(2012Pattern)

Time: 2Hours

Max. Marks : 50

Instructions to the candidates:

- 1) Attempt Four Questions : Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 to 6, Q. No. 7 or 8
- 2) Figures to the right side indicate full marks.
- 3) Use of Electronic non-programmable Calculator is allowed.
- 4) Assume Suitable data if necessary.

Q1) a) Examine the following system of equations for consistency and solve it, if consistent. [04]

$$\begin{aligned}4x - 2y + 6z &= 8, \\ x + y - 3z &= -1, \quad 15x - 3y + 9z = 21\end{aligned}$$

b) Examine the following vectors for Linear dependence. [04]

Find the relation between them, if dependent.
(2,-1,3,2), (1,3,4,2) and (3,-5,2,2)

c) If $2 \cos \phi = x + 1/x$, $2 \cos \psi = y + 1/y$ [04]

Prove that, $x^p y^q + 1/x^p y^q = 2 \cos (p\phi + q\psi)$

OR

Q2) a) Use De Moivres theorem, to solve the equation [04]

$$X^7 + X^4 + 1 (X^3 + 1) = 0$$

b) If $(1 + ai)(1 + bi) = p + iq$, then prove that, [04]

1. $p \tan [\tan^{-1}a + \tan^{-1}b] = q$

2. $(1 + a^2)(1 + b^2) = p^2 + q^2$

c) Reduce the following matrix A to its normal form and hence find its rank, where [04]

$$A = \begin{vmatrix} 2 & -3 & 4 & 4 \\ 1 & 1 & 1 & 2 \\ 3 & -2 & 3 & 6 \end{vmatrix}$$

Q3) a) Test convergence of the series (Any One) [04]

1. $\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$

$$2. \frac{1*2}{3^2*4^2} + \frac{3*4}{5^2*6^2} + \frac{5*6}{7^2*8^2} + \dots$$

b) Expand $40 + 53(x-2) + 19(x-2)^2 + 2(x-2)^3$
In ascending powers of x

[04]

c) If $y = x^n \log x$ then, prove that

[04]

$$Y_{n+1} = \frac{n!}{x}$$

OR

Q4) a) Solve any one

[04]

1) Evaluate $\lim_{x \rightarrow \infty} (\cot x)^{\sin x}$

2) Find the values of a and b such that,

$$\lim_{x \rightarrow \infty} \frac{a \cos x - a + b x^2}{x^4} = \frac{1}{12}$$

b) Prove that,

[04]

$$e^x \tan x = x + x^2 + \frac{5x^3}{6} + \frac{x^4}{2} + \dots$$

c) If $Y = \frac{x}{(x+1)^4}$ find Y_n

[04]

Q5) Solve any Two of the following

[13]

1. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for $u = \tan^{-1} \left[\frac{y}{x} \right]$

2. If $x = u \tan v$, $y = u \sec v$

$$\text{Prove that, } \left(\frac{\partial u}{\partial x} \right)_y \left(\frac{\partial v}{\partial x} \right)_y = \left(\frac{\partial u}{\partial y} \right)_x \left(\frac{\partial v}{\partial y} \right)_x$$

3. If $u = \frac{x^3 + y^3}{y\sqrt{x}} + \frac{1}{x^7} \sin^{-1} \left(\frac{x^2 + y^2}{2xy} \right)$

Then, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \quad \text{At point (1,1)}$$

OR

Q6) Solve any Two of the following [13]

A) If

$u = (x^2 - y^2) f(xy)$ then show that

$$u_{xx} + u_{yy} = (x^4 - y^4) f''(xy)$$

B) Verify Eulers theorem for homogenous functions

$$F(x, y, z) = 3x^2 yz + 5xy^2 z + 4z^4$$

C) If $x = u + v + w$, $y = uv + uw + vw$,
 $z = uvw$

and F is function of x,y,z then prove that,

$$x \frac{\partial F}{\partial x} + 2y \frac{\partial F}{\partial y} + 3z \frac{\partial F}{\partial z} = u \frac{\partial F}{\partial u} + v \frac{\partial F}{\partial v} + w \frac{\partial F}{\partial w}$$

Q7) a) If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ [04]

Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

b) Examine for functional dependence for

$$u = x + y + z, \quad v = x^2 + y^2 + z^2, \quad w = x^3 + y^3 + z^3 - 3xyz \quad [04]$$

c) Find the extreme values of [05]

$$f(x, y) = x^3 + y^3 - 3axy, \quad a > 0$$

OR

Q8) a) If $u^2 + xv^2 = x + y$ and $v^2 + yu^2 = x - y$ find $\frac{\partial v}{\partial y}$ [04]

b) The resistance R of a circuit was calculated using the formula [04]

$I = E / R$. If there is an error of 0.1Amp in reading I and 0.5 Volts in E, find the corresponding percentage error in R
When I = 15 Amp and E= 100 Volts

c) Divide 24 into three parts such that, the continued product of the first, square of the second and cube of the third may be maximum. Use Lagrange's method. [05]