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<b>Seat No.</b>	
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**[4756]-101**

**F.E. (I Sem.) EXAMINATION, 2015**

**ENGINEERING MATHEMATICS—I**

**(2012 COURSE)**

**Time : Two Hours**

**Maximum Marks : 50**

**N.B. :—** (i) Attempt *All* questions.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of logarithmic tables, electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (A) Find the eigen values and eigen vector corresponding to minimum eigen value for the matrix : [4]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

P.T.O.

(B) Determine the value for  $\lambda$  for which the equations :

$$3x_1 + 2x_2 + 4x_3 = 3,$$

$$x_1 + x_2 + x_3 = \lambda,$$

$$5x_1 + 4x_2 + 6x_3 = 15,$$

are consistent. Find also the corresponding solution. [4]

(C) If  $z_1, z_2, z_3$  are the vertices of an isosceles triangle right angled at  $z_2$ , prove that : [4]

$$z_1^2 + z_3^2 + 2z_2^2 = 2z_2(z_1 + z_3).$$

*Or*

2. (A) If [4]

$$2 \cos \theta = x + \frac{1}{x},$$

prove that :

$$2 \cos r\theta = x^r + \frac{1}{x^r}.$$

(B) If  $Y = \log \tan x$ , prove that : [4]

(i)  $\sinh ny = \frac{1}{2}(\tan^n x - \cot^n x),$

(ii)  $2 \cosh ny \operatorname{cosec} 2x = \cosh(n+1)y + \cosh(n-1)y.$

- (C) Examine for linear dependence or independence for the given vectors and if dependence, find the relation between them : [4]

$$X_1 = (1, -1, 2, 2),$$

$$X_2 = (2, -3, 4, -1),$$

$$X_3 = (-1, 2, -2, 3).$$

3. (A) Test convergence of the series (any one) : [4]

(i) 
$$\sum_{n=1}^{\infty} \left( \frac{2n+1}{3n+4} \right) 5^n$$

(ii) 
$$\frac{1}{1+\sqrt{2}} + \frac{2}{1+2\sqrt{3}} + \frac{3}{1+3\sqrt{4}} + \dots$$

- (B) Prove that : [4]

$$\sin x \cosh x = x + \frac{1}{3}x^3 - \frac{1}{30}x^5 - \dots$$

- (C) Find  $n$ th derivative of : [4]

$$e^{2x} \sinh 3x \cos 4x.$$

Or

4. (A) Solve any one : [4]

(i) 
$$\lim_{x \rightarrow a} (x-a)^{(x-a)}$$

(ii) 
$$\lim_{x \rightarrow \pi/2} (\sec x - \tan x).$$

(B) Using Taylor's theorem, expand :

$$2x^3 + 3x^2 - 8x + 7$$

in powers of  $x - 2$ . [4]

(C) If  $y = e^{\tan^{-1} x}$ , then show that : [4]

$$(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n - 1)y_{n-1} = 0.$$

5. Solve any two :

(A) Find the value of  $n$  for which : [6]

$$z = A e^{-gx} \sin(nt - gx),$$

satisfies the partial differential equation :

$$\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2}.$$

(B) If

$$u = \frac{x^4 + y^4}{x^2 y^2} + x^6 \tan^{-1} \left[ \frac{x^2 + y^2}{x^2 + 2xy} \right],$$

find the value of :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

at  $x = 1, y = 2$ . [7]

(C) If  $z = f(u, v)$  and [6]

$$u = \log(x^2 + y^2), v = \frac{y}{x},$$

show that :

$$x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = (1 + v^2) \frac{\partial z}{\partial v}.$$

Or

6. Solve any two :

(A) If [6]

$$x = \frac{\cos \theta}{r}, y = \frac{\sin \theta}{r},$$

find the value of :

$$\left(\frac{\partial x}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial x}\right)_y + \left(\frac{\partial y}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial y}\right)_x.$$

(B) If [7]

$$u = \sin^{-1} \sqrt{\frac{x^2 + y^2}{x + y}},$$

show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} \tan u [\tan^2 u - 1].$$

(C) If  $z = f(u, v)$  and  $u = x \cos t - y \sin t$ ,  $v = x \sin t + y \cos t$ , where  $t$  is a constant, prove that : [6]

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = u \frac{\partial z}{\partial u} + v \frac{\partial z}{\partial v}.$$

7. (A) If  $u^3 + v^3 = x + y$ ,  $u^2 + v^2 = x^3 + y^3$ , find  $\frac{\partial(u, v)}{\partial(x, y)}$ . [4]

(B) Determine whether the following functions are functionally dependent. If functionally dependent, find the relation between them : [4]

$$u = \sin x + \sin y, v = \sin(x + y).$$

- (C) Examine maxima and minima of the following function and find their extreme values : [5]

$$(x^2 + y^2 + 6x + 12).$$

*Or*

8. (A) If  $x = u + v$ ,  $y = v^2 + w^2$ ,  $z = w^3 + u^3$ , show that : [4]

$$\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}.$$

- (B) If  $e^z = \sec x \cos y$  and errors of magnitude  $h$  and  $-h$  are made in estimating  $x$  and  $y$ , where  $x$  and  $y$  are found to be  $\frac{\pi}{3}$  and  $\frac{\pi}{6}$  respectively, find the corresponding error in  $z$ . [5]

- (C) Find the minimum distance from origin to the plane : [4]

$$3x + 2y + z = 12.$$