

Total No. of Questions : 8]

SEAT No. :

P2021

[Total No. of Pages : 4

**F.E.**

**ENGINEERING MATHEMATICS - I**

**(2012 Pattern)**

*Time : 2 Hours]*

*[Max. Marks : 50*

*Instructions to the candidates:*

- 1) *Attempt four questions : Q.no. 1 or 2, Q.no. 3 or 4, Q.no. 5 or 6 and Q.no. 7 or 8.*
- 2) *Figures to the right indicate full marks.*
- 3) *Assume suitable data if necessary.*
- 4) *Neat diagram must be drawn wherever necessary.*
- 5) *Use of electronic non-programmable calculator is allowed.*

**Q1) a)** Show that the system of equations. **[4]**

$$3x + 4y + 5z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$5x + 6y + 7z = \gamma$$

is consistent only if  $\alpha$ ,  $\beta$  and  $\gamma$  are in arithmetic progression.

**b)** Verify Cayley - Hamilton theorem for A and hence find  $A^{-1}$ . **[4]**

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

**c)** If  $\alpha = 1 + i$ ,  $\beta = 1 - i$  and  $\cot Q = x + 1$  then prove that

$$\frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta} = \sin nQ \operatorname{cosec}^n Q \quad \text{[4]}$$

OR

**Q2) a)** Examine whether the following vectors are linearly dependent or independent. If dependent, find the relation between them. **[4]**

$$X_1 = [1, 2, 3], X_2 = [3, -2, 1], X_3 = [1, -6, -5]$$

**P.T.O.**

b) Show that [4]

$$\log \left[ \frac{\sin(x+iy)}{\sin(x-iy)} \right] = 2i \tan^{-1}(\cot x \tanh y).$$

c) Prove that  $\tan^{-1} \left[ i \left( \frac{x-a}{x+a} \right) \right] = -\frac{i}{2} \log \left( \frac{a}{x} \right)$ . [4]

**Q3)** a) Test the convergence of the series (Any one) [4]

i)  $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} + \dots + \frac{n+1}{n^3} + \dots$

ii)  $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots$

b) Expand  $x^3 + 7x^2 + x - 6$  in powers of  $(x - 3)$  [4]

c) If  $y = \log(x + \sqrt{x^2 + 1})$ , prove that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0. \quad [4]$$

OR

**Q4)** a) Solve any one [4]

i) Evaluate  $\lim_{x \rightarrow \pi/2} (\cos x)^{\cos x}$ .

ii) If  $\lim_{x \rightarrow 0} \frac{\sin 2x + \rho \sin x}{x^3}$  is finite, find the value of  $\rho$  and hence evaluate the limit.

b) Prove that  $\log(1 + \tan x) = x - \frac{x^2}{2} + \frac{2x^3}{3} \dots$  [4]

c) Find  $n^{\text{th}}$  derivative of  $y = \frac{x}{(x-1)(x-2)(x-3)}$ . [4]

**Q5)** Solve any two of the following :

- a) Find the value of  $n$  such that  $u = x^n (3\cos^2 y - 1)$  satisfies the partial differential equation. [6]

$$\frac{\partial}{\partial x} \left( x^2 \frac{\partial u}{\partial x} \right) + \frac{1}{\sin y} \frac{\partial}{\partial y} \left( \sin y \frac{\partial u}{\partial y} \right) = 0$$

- b) If  $x = r \cos \theta$ ,  $y = r \sin \theta$

then prove that

[6]

i) 
$$\left( \frac{\partial y}{\partial r} \right)_x \left( \frac{\partial y}{\partial r} \right)_\theta = 1$$

ii) 
$$\left( \frac{\partial x}{\partial \theta} \right)_r = r^2 \left( \frac{\partial \theta}{\partial x} \right)_y$$

- c) If  $u = x^8 f\left(\frac{y}{x}\right) = \frac{1}{y^8} \phi\left(\frac{x}{y}\right)$ ,

[7]

then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 64u$$

OR

**Q6)** Solve any two of the following :

- a) If  $u = \cos^{-1} \left[ \frac{x^3 y^2 + 4y^3 x^2}{\sqrt{x^4 + 6y^4}} \right]$

[7]

find the value of

i) 
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

ii) 
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

b) If  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ .

prove that

[6]

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

c) If  $f(x, y) = 0$ ,  $\phi(x, z) = 0$

[6]

then prove that

$$\frac{\partial \phi}{\partial x} \frac{\partial f}{\partial y} \frac{dy}{dz} = \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial z}$$

Q7) a) If  $x = e^v \sec u$ ,  $y = e^v \tan u$ , find  $\frac{\partial(u, v)}{\partial(x, y)}$ .

[4]

b) Examine for functional dependence for

[4]

$$u = \frac{x}{y-z}, \quad v = \frac{y}{z-x}, \quad w = \frac{z}{x-y}$$

c) Find extreme values of  $f(x, y) = x^3 + y^3 - 3axy$ ,  $a > 0$

[5]

OR

Q8) a) If  $x = \cos \theta - r \sin \theta$ ,  $y = \sin \theta + r \cos \theta$  find  $\frac{\partial r}{\partial x}$ .

[4]

b) The resonant frequency in a series electrical circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

If the measurement in L and C are in error by 2% and -1% respectively. Find the percentage error in f.

[4]

c) Use Lagrange's method to find stationary value of  $u = \frac{x^2}{a^3} + \frac{y^2}{b^3} + \frac{z^2}{c^3}$  where  $x + y + z = 1$ .

[5]

