Seat No.	

F.E. (Semester – II) Examination, 2014
ENGINEERING MATHEMATICS – II (Old)
(2008 Course)

Time : 3 Hours   Max. Marks : 100

Instructions : *1) In Section – **I** solve Q. **1** or Q. **2,** Q. **3** or Q. **4**, Q. **5** or Q. **6**. In Section – **II** solve Q. **7** or Q. **8**, Q. **9** or Q. **10**, Q. **11** or Q. **12**.*
*2) Neat diagrams must be drawn **wherever** necessary.*
*3) Figures to the **right** indicate **full** marks.*
*4) **Use** of non-programmable electronic pocket calculator is **allowed**.*
*5) Assume suitable data, if **necessary**.*

SECTION – I

1. a) Form the differential equation whose general solution is $y = c_1e^x + c_2e^{-x} + 3x$. **6**

   b) Solve **any two** : **10**

   i) $\dfrac{dy}{dx} = \dfrac{x^2 + 2y^2}{xy}$

   ii) $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y}{\sin x + x\cos y} = 0$

   iii) $\dfrac{dy}{dx} - xy = -y^3e^{-x^2}$

OR

2. a) Form the differential equation whose general solution is $y = (c_1 + c_2t)e^t$. **6**

   b) Solve **any two** : **10**

   i) $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

   ii) $\dfrac{dy}{dx} + \dfrac{y}{1-x} = x^2 - x$

   iii) $\dfrac{dy}{dx} = \dfrac{x+y+3}{3x+3y-3}$

3. Solve **any three** : **18**

   i) A body at temperature 90°C is placed in a room whose temperature is 30°C and cools to 50°C in 6 minutes. Find temperature after a further interval of 6 minutes.

   ii) A resistance of 120 Ω and inductance of 0.6 henry are connected in series with battery 30 volts. Find current in a circuit as a function of t.

iii) Find the orthogonal trajectories of the curves given by $y = 4ax^2$.

iv) A body start moving from rest is apposed by force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are displacement and velocity of particle at that instant, show that the velocity of particle is given by $v^2 = \frac{c}{2b^2}\left(1-e^{-2bx}\right)-\frac{cx}{b}$.

OR

4. Solve **any three**. **18**

i) A pipe 10 cm in diameter contains steam at 100°C it is covered with asbestos 5 cm thick for which k = 0.006 and the outside temperature is at 30°C. Find the amount of heat lost per hour from a meter long pipe.

ii) Radium decomposes at the rate proportional to the amount present. If 5% of the origin amount disappear in 50 years. How much remains after 75 years ?

iii) Equation of L-R circuit is given by $L\frac{dI}{dt}+RI=10\sin t$

i = 0 at t = 0, express i as function of t.

iv) A metal ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintain at 50°C, if temperature of ball is reduced to 70°C in 5 minute. Find the time at which temperature of ball is 60°C.

5. a) Obtain the Fourier series for the periodic function **9**

$$f(x)=\begin{cases}\sin x & 0\le x\le \pi\\ 0 & \pi\le x\le 2\pi\end{cases}$$

Hence deduce that $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....=\frac{1}{2}$

b) If $U_n = \int_0^{\pi/4}\sec^n\theta\, d\theta$, prove that $U_n=\frac{\left(\sqrt{2}\right)^{n-2}}{n-1}+\frac{n-2}{n-1}U_{n-2}$, hence evaluate U_4. **7**

OR

6. a) Compute first two harmonics of the Fourier series of f(x) given in the table **8**

x	0	60°	120°	180°	240°	300°
f (x)	1	1.4	1.9	1.7	1.5	1.2

b) Evaluate $\int_0^{\infty} e^{-\sqrt{x}}\, x^{1/4}\, dx$. **4**

c) Evaluate $\int_0^{\infty}\frac{x^4\left(1-x^2\right)}{(1+x)^{12}}dx$ **4**

SECTION – II

7. a) Trace the following curves (**any two**): **8**

   i) $xy^2 = a^2 (a - x)$

   ii) $r^2 = a^2 \cos 2\theta$

   iii) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

   b) Verify rule of DUIS for $I = \int_0^{\pi/2} e^{ax} dx$. **5**

   c) Find the perimeter of the cardioid $r = a[1 + \cos\theta]$. **4**

OR

8. a) Trace the following curves (**any two**) : **8**

   i) $x = a\cos^3 t$, $y = b\sin^3 t$

   ii) $r = a\cos 3\theta$

   iii) $y^2 (2a - x)x^3$

   b) Show that $\int_0^{\infty} e^{-x^2 - 2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2} [1 - \text{erf}(b)]$. **5**

   c) Find the length of the arc of the curve $y = c\cosh[x/c]$ measured from the vertex to any point (x, y) and show that $s^2 = y^2 - c^2$. **4**

9. a) Find the equation of the sphere which passes through the point (3, 1, 2) and meets X0Y-plane in a circle of radius 3 units with the centre at (1, – 2, 0). **6**

   b) Find the semi-vertical angle and the equation of right circular cone having its vertex at the (0, 0, 0) and passing through the circle $x^2 + z^2 = 25$ and $y = 4$. **5**

   c) Find the equation of right circular cylinder whose axis is the line $2(x - 1) = y + 2 = z$ and radius is 2. **6**

OR

10. a) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. **6**

   b) Obtain the equation of the right circular cone which passes through (1, 3, 4) with vertex (2, 2, 1) and axis parallel to the line $\frac{x+1}{2} = \frac{y-1}{-2} = \frac{z-2}{3}$. **6**

   c) Find the equation of the right circular cylinder described on the circle through (a, 0, 0), (0, a, 0) (0, 0, a). **5**

11. Solve **any two**.

   a) Evaluate $\int_0^{\sqrt{a^2 - y^2}} \left[\sin\frac{\pi}{a^2}(a^2 - x^2 - y^2)\right] dxdy$. **8**

b) Find the position of the centroid of the area bounded by the curve $y^2 (2a - x) = x^2$ and the asymptote. **8**

c) Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and paraboloid $z = x^2 + y^2$. **8**

OR

12. Solve **any two**.

a) Find the area of the loop of the curve $a^4 y^2 = x^5 [2a - x]$. **8**

b) Evaluate $\iiint \left[x^2y^2 + y^2z^2 + z^2x^2\right] dxdydz$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$. **8**

c) Find the moment of inertia about the x-axis of the area enclosed by the lines $x = 0, \frac{x}{a} + \frac{y}{b} = 1$. **8**
