UNIVERSITY OF PUNE

[4361-7]

F.E. Examination 2013

Engineering Mathematics -II

(2008 pattern)

Time-Three hours **Maximum Marks-100**

[Total No. of Question=12] **[Total no. of printed pages= 5]**

Instructions:

(1)In Section -I Solve Q.1 or Q.2,Q.3 or Q.4,Q.5 or Q.6. In Section-II Solve Q.7 or Q.8,Q.9 or Q.10,Q.11 or Q.12

(2)Neat diagrams must be drawn wherever necessary.

(3)Figures to the right indicate full marks.

(4)Use of electronic pocket calculator is allowed.

(5)Assume suitable data wherever necessary.

SECTION-I

Q.1 (a)Form the differential equation whose general solution is

$(X-A)^2+(Y-B)^2=16$ where A and B are arbitrary constants . (6)

(b)Solve **any two**. (10)

(i) $y(x^2 y+e^x)dx-e^x dy=0$

(ii) $\frac{dy}{dx}=\frac{x+y+1}{2x+2y+3}$

(iii) $\frac{dy}{dx}+y\cot x=sin2x$

OR

Q.2 (a)By eliminating arbitrary constants a & b find the differential equation whose general solution is $y=\log\cos(x-a)+b.$ (6)

(b)Solve any two. (10)

(i) $\frac{dy}{dx}=e^{x-y}(e^x-e^y)$

(ii) $\frac{dy}{dx}=\frac{\tan y-2xy-y}{x^2-x\tan^2 y+sec^2 y}$

(iii) $(y^4-2x^3 y)dx+(x^4-2xy^3)dy=0$

Q.3 Solve any three. (18)

(a)If the temperature of the body drops from $100^0 C$ to $60^0 C$ in one minute,the temperature of surrounding being $20^0 C$,what will be the temperature of the body after two minutes?

(b)In a circuit containing inductance L,resistance R and voltage E,the current I is given by $E=RI+L\frac{dI}{dt}$ .Given L=320H , R=125Ω and E =250 volts, I being zero when t=0. Find the time that elapses before current reaches half of its theoretical maximum.

(c)A particles is moving in a straight line with an acceleration $k\left[x+\frac{a^4}{x^4}\right]$,directed towards origin. If it starts from rest at a distance 'a' from origin, prove that it will arrive at origin at the end of time $\frac{\pi}{4\sqrt{k}}$.

(d)Find orthogonal trajectories of the family of curves given by $x^2+2y^2=c^2$ where c is arbitrary constant.

OR

Q.4 Solve any three. (18)

(a)A pipe 10 cm in diameter contains steam at $200^0 C$ .It is protected with a covering 5cm thick, for which k=0.12 and outside surface is at $50^0 C$ .Find the temperature half way through the covering under steady state conditions.

(b)A body of mass m falls from rest under the influence of gravity and a retarding force, due to air resistance, proportional to instantaneous velocity of the body. Find velocity and distance described as a function of time.

(c)The charge 'Q' on the plate of a condenser of capacity 'C' charged through a resistance 'R' by a steady voltage 'V' satisfies the differential equation.

$R\frac{dQ}{dt}+\frac{Q}{C}=V$ .If Q=0 at t=0, show that $Q=CV[1-e^{-t/RC}]$ .Find the current flowing into the circuit.

(d)The amount x of a substance in a certain chemical reaction at time t is given by

$\frac{dx}{dt}+\frac{x}{10}=2-1.5e^{-t/10}$ .If at t=0, x=0.5, find x at t=10.

Q.5 (a)Find the Fourier series to represent the function $f(x)=x^2$,in the interval $-\pi<x<\pi$ and $f(x+2\pi)=f(x)$ for all x.

Deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....=\frac{\pi^2}{6}$. (9)

(b)If $I_n=\int_0^{\frac{\pi}{4}}\cos^{2n}x\,dx$, prove that $I_n=\frac{1}{n2^{n-1}}+\frac{2n-1}{2n}I_{n-1}$

Hence evaluate $\int_0^{\frac{\pi}{4}}\cos^6 x\,dx$. (7)

OR

Q.6 (a)Find the Fourier series upto first harmonics to represent f(x) in the interval $(0,2\pi)$ from the tabulated values of x & f(x) given below. (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(b)Evaluate ; $\int_0^{\infty}\sqrt{x}\,e^{-\sqrt{x}}\,dx$. (4)

(c) $\int_3^7 (x-3)^{1/4}(7-x)^{1/4}\,dx$. (4)

SECTION-II

Q.7 (a)Trace the following curves.(any two). (8)

(i) $3ay^2 = x(x-a)^2$

(ii) $r = a(1+\cos\theta)$

(iii) $x = t^2, y = t - \frac{t^3}{3}$

(b)Using DUIS evaluate. (5)

$$\int_0^\infty \frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right)dx$$

(c)Using proper rectification formula, find the circumference of the circle of radius a. (4)

OR

Q.8 (a)Trace the following curves (any two). (8)

(i) $a^2 y^2 = x^2(2a-x)(x-a)$

(ii) $x = a(t+\sin t), y = a(1-\cos t)$

(iii) $r = a + b\cos\theta$ for a > b

(b)Show that $\int_0^\infty e^{-(x+a)^2}dx = \frac{\sqrt{\pi}}{2}[1 - erf(a)]$ (4)

(c)Find the whole length of the loop of the curve $9y^2 = (x+7)(x+4)^2$ (5)

Q.9 (a)A sphere of constant radius r passes through the origin and cuts the axes in the points A,B,C. Find the locus of the foot of perpendicular from origin to the plane ABC. (6)

(b)Find the equation of the right circular cone with vertex at (1,2,-3), semivertical angle $\cos^{-1}(\frac{1}{\sqrt{3}})$ and axis is the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{-1}$ (6)

(c)Find the equation of the right circular cylinder of radius 2,whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (5)

OR

Q.10 (a)Find the equations of tangent planes to the sphere $x^2+y^2+z^2=9$ which pass through the line $x+y=6, x-2z=3$. (6)

(b)Find the equation of right circular cone which has its vertex at the point (0,0,10)and whose intersection with the xoy plane is circle of diameter 10. (6)

(c)Find the equation of right circular cylinder of radius 2 where axis pass through (1,2,3) and has direction cosines proportional to 2,-3,6. (5)

Q.11 Solve ant two. (8)

(a)Evaluate $\int\int_R \sqrt{xy(1-x-y)}\,dx\,dy$ where R is the area bounded by x=0, y=0 and x+y=1. (8)

(b)Change the order of integration in double integral . (8)

$$\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dx\,dy$$

(c)Find the area common to the circles. (8)

$x^2+y^2=a^2$ And $x^2+y^2=2ax$

OR

Q.12 Solve any two.

(a)Express the following as a single term integral and evaluate: (8)

$$\int_0^{\frac{a}{\sqrt{2}}}\int_0^{x} \cos k(x^2+y^2)\,dx\,dy+\int_{\frac{a}{\sqrt{2}}}^{a}\int_0^{\sqrt{a^2-x^2}} \cos k(x^2+y^2)\,dx\,dy$$

(b)Evaluate $\int_0^\infty\int_0^\infty\int_0^\infty \frac{dxdydz}{(1+x^2+y^2+z^2)^2}$. (8)

(c)If the density at any point of the curve $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ varies as its distance from the x-axis,find the distance of its C.G. of arc from the x-axis.(8)