[4656] – 11

Seat No.	

F.E. (Semester – I) Examination, 2014
ENGG. MATHEMATICS – I
(Old) (2008 Course)

Time : 3 Hours Max. Marks : 100

Instructions : *1) Answer **3** questions from Section **I** and **3** questions from Section **II**.*
*2) Answers to the **two** Sections should be written in **separate** books.*
*3) Black figures to the **right** indicate **full** marks.*
*4) **Use** of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is **allowed**.*
*5) Assume suitable data, **if necessary**.*

SECTION – I

1. a) Reduce the matrix $A=\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ to its normal form and hence find it's rank. **5**

b) Determine the values of a and b for which the system of equations,

$2x + 3y + 5z = 9$

$7x + 3y – 2z = 8$

$2x + 3y + az = b$ have

i) Unique solution ii) Infinite solutions iii) No solution. **6**

c) Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ and hence find A^{-1}. **6**

OR

2. a) Find eigen values and eigen vectors for $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. **6**

b) Examine the vectors $x_1 = (3, 1, – 4)$, $x_2 = (2, 2, – 3)$, $x_3 = (0, – 4, 1)$ for Linear dependence or independence. **6**

c) If $Y=\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ find (x_1, x_2, x_3) corresponding to (2, 3, 0) in y. **5**

P.T.O.

3. a) Show that $\dfrac{1}{(x+iy)^2}+\dfrac{1}{(x-iy)^2}$ is real. **5**

b) If $\tan(\theta+i\phi) = \cos\alpha + i\sin\alpha$

Prove that 1) $\theta=\dfrac{n\pi}{2}+\dfrac{\pi}{4}$

2) $\phi=\dfrac{1}{2}\log\tan\left(\dfrac{\pi}{4}+\dfrac{\alpha}{2}\right)$. **6**

c) Show that $\left(1+i\sqrt{3}\right)^{1/3}+\left(1-i\sqrt{3}\right)^{1/3}=2^{4/3}\cos\dfrac{(6K+1)\pi}{g}$

K = 0, 1, 2. **5**

OR

4. a) If two adjacent vertices of a square are – 2 and 2i, find the complex number representing other vertices. **6**

b) Show that i^i is wholly real and all its values form Geometric progression. **5**

c) Find all the roots of the equation $x^4 + 1 = 0$. **5**

5. a) If $y = \cos^4 x$ find y_n. **6**

b) If $y=\left(x+\sqrt{x^2-1}\right)^m$ prove that $(x^2-1)\,y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)\,y_n = 0$. **6**

c) Discuss convergence or divergence **(any one)** : **5**

1) $\sum\limits_{n=1}^{\infty}\dfrac{1}{\sqrt{n}+\sqrt{n+1}}$

2) $\sum\limits_{n=1}^{\infty}\dfrac{2^2.4^2\ldots(2^n)^2}{3.4.5.6\ldots(2n+1)(2n+2)}$.

OR

6. a) If $y=\dfrac{x^2}{(x+2)(2x+3)}$ then find y_n. **6**

b) If $y=e^{a\sin^{-1}x}$ prove that $(1-x^2)\,y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)\,y_n = 0$. **6**

c) Discuss convergence or divergence of **(any one)**. **5**

1) $\dfrac{1}{2\sqrt{1}}+\dfrac{x^2}{3\sqrt{2}}+\dfrac{x^4}{4\sqrt{3}}+\dfrac{x^6}{5\sqrt{4}}+\ldots$ 2) $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n\sqrt{n}}$.

SECTION – II

7. A) If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$ Prove that $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ and conversely. **5**

B) Using Taylor's theorem, expand $2x^3 + 7x^2 + x - 6$ in powers of '$x - 2$'. **5**

C) Solve **any one** of the following : **6**

i) $\lim_{x \to 1} \frac{x}{x-1} - \frac{1}{\log x}$

ii) Find values of a, b and c if $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$.

OR

8. A) Show that $\tan^{-1}\left[\frac{\sqrt{1+x^2-1}}{x}\right] = \frac{1}{2}\left[x - \frac{x^3}{3} + \frac{x^5}{5} \ldots\right]$. **5**

B) Using Taylor's theorem, find expansion of $\tan(x + \pi/4)$ in ascending powers of 'x' upto term in x^4, Find approximately value of tan 43°. **5**

C) Solve **any one** of the following : **6**

i) $\lim_{x \to \infty} \left(\frac{ax+1}{ax-1}\right)^x$ ii) $\lim_{x \to 0} \frac{e^{2x} - (1+x)^2}{x\log(1+x)}$

9. Solve **any two** of the following : **16**

A) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

B) If $u = \sin^{-1}(x^3 + y^3)^{2/5}$, then find the value of $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy}$.

C) If $z = f(x, y)$ where $x = e^u + e^{-v}$ $y = e^{-u} - e^v$ show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

OR

10. Solve **any two** of the following : **16**

A) If $x^2 = a\sqrt{u} + b\sqrt{v}$, $y^2 = a\sqrt{u} - b\sqrt{v}$. Show that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$.

B) If $ax^2 + by^2 + cz^2 = 1$, and $lx + my + nz = 0$. Prove that $\frac{dx}{bny - cmz} = \frac{dy}{clz - anx} = \frac{dz}{amx - bly}$.

C) If $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2} + \frac{x^2 y}{x + y}$. Find $x\frac{\partial T}{\partial x} + y\frac{\partial T}{\partial y}$.

11\. A) If $x + y = 2e^{\theta} \cos\phi$, $x - y = 21^{0} e^{\theta} \sin \phi$ verify $JJ^{1} = 1$. **6**

B) Verify whether following functions are functionally dependent, if so find relation between them.

$$u=\frac{x-y}{x+y} \qquad v=\frac{xy}{(x+y)^2}.$$ **6**

C) Find maximum and minimum values of the function $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **6**

OR

12\. A) Find possible percentage error in computing parallel resistance 'r' of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$, if r_1, r_2, r_3 are each in error by 1.2%. **6**

B) If $x = u+v+w$, $y = u^2 + v^2+ w^2$, $z = u^3+ v^3 + w^3$ show that $\frac{\partial u}{\partial x}=\frac{vw}{(u-v)(u-w)}$. **6**

C) Show that stationary value of $a^3x^2+ b^3y^2+c^3z^2$, where $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ is given by

$$x=\frac{a+b+c}{a} \quad y=\frac{a+b+c}{b} \quad z=\frac{a+b+c}{c}$$ **6**

———————