Total No. of Questions : 12]

SEAT No. :

P559

[Total No. of Pages : 5

[4456]-1
F.E. (Semester - I)
ENGG. MATHEMATICS - I
(2008 Course)

Time : 3 Hours] *[Max. Marks : 100*

Instructions to the candidates:

1) *Answer 3 questions from Section I and 3 questions from Section II.*
2) *Answers to the two sections should be written in separate books.*
3) *Figures to the right indicate full marks.*
4) *Use of logarithmic tables, slide rule, mollier charts, electronic pocket calculator and steam tables is allowed.*
5) *Assume suitable data, if necessary.*

SECTION - I

Q1) a) Reduce the following matrix to its normal form and hence find its rank,

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ **[5]**

b) Investigate for what values of λ and μ the system of simultaneous equations **[6]**

$x + y + z = 6$; $x + 2y + 3z = 10$

; $x + 2y + \lambda z = \mu$

have

i) No solution

ii) A unique solution

iii) An infinite number of solutions.

c) Verify Cayley - Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and use it to find A^{-1}. **[7]**

OR

P.T.O.

Q2) a) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$. **[6]**

b) Examine for linear dependance or independance of vectors (2, –1, 3, 4), (1, 3, 4, 2) and (3, –5, 2, 2). Find a relation between them if dependent. **[6]**

c) Determine the values of a, b, c when $A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$ is orthogonal. **[6]**

Q3) a) If $x_r=\cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$, then show that $x_1.x_2.x_3..........\infty=-1$. **[5]**

b) Find z if $|z+i|=|z|$ and $\arg\left(\frac{z+i}{z}\right)=\frac{\pi}{4}$ **[5]**

c) If $\sin(\alpha+i\beta)=x+iy$; then prove that **[6]**

i) $\frac{x^2}{\cosh^2\beta}+\frac{y^2}{\sinh^2\beta}=1$

ii) $\frac{x^2}{\sin^2\alpha}-\frac{y^2}{\cos^2\alpha}=1$

OR

Q4) a) If z_1, z_2, z_3 represents vertices of an equilateral triangle, then prove that

$z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$ **[5]**

b) Find all the values of $(1-i\sqrt{3})^{1/4}$ **[6]**

c) Prove that i^i is wholly real and find its principal value of logarithm. **[5]**

Q5) a) If $y=\sin px+\cos px$, then prove that $y_n=p^n[1+(-1)^n\sin(2px)]^{1/2}$. **[5]**

b) If $y=\sin^{-1}x$, then prove that, $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ **[5]**

c) Discuss convergence or divergence (Any one) **[6]**

i) $\frac{2}{1}+\frac{3}{8}+\frac{4}{27}+\frac{5}{64}+.........$

ii) $\sum_{n=1}^{\infty}\frac{3.6.9..........3n}{7.10.13.......(3n+4)}$

OR

Q6) a) Find n^{th} derivative of $y=\frac{x^3}{(x-1)(x-2)}$. **[5]**

b) If $x=\tan(\log y)$; then prove that,

$(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$ **[5]**

c) Discuss the convergence or divergence (Any one) **[6]**

i) $\log\left(\frac{1}{2}\right)-\log\left(\frac{2}{3}\right)+\log\left(\frac{3}{4}\right)-\log\left(\frac{4}{5}\right)+..........$

ii) $\sum_{n=1}^{\infty}\frac{x^n}{a+\sqrt{n}}$

<u>SECTION - II</u>

Q7) a) Expand $\log\left[\frac{1+e^{2x}}{e^x}\right]$ up to x^6. **[5]**

b) Using Taylor's theorem find $\sin(30°,30^1)$ **[5]**

c) Solve : (Any one) **[6]**

i) Find a and b, if $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$

ii) Evaluate $\lim_{n\to\infty}\left[\cos\left(\frac{\theta}{n}\right)\right]^{n^2}$

OR

Q8) a) Using Taylor's theorem express $7+(x+2)+3(x+2)^3+(x+2)^4$ in ascending powers of x. **[5]**

b) Expand $\sinh^{-1} x$ in ascending powers of x. **[5]**

c) Solve : (Any one) **[6]**

i) Evaluate $\lim_{x\to 0}\left[\dfrac{2^x+3^x}{2}\right]^{\frac{1}{x}}$.

ii) Evaluate $\lim_{x\to 0}\dfrac{\sin x\sin^{-1}x-x^2}{x^6}$

Q9) Solve : (Any two) **[16]**

a) If $v=(1-2xy+y^2)^{-1/2}$ prove that

i) $x\dfrac{\partial v}{\partial x}-y\dfrac{\partial v}{\partial y}=y^2v^3$

ii) $\dfrac{\partial}{\partial x}\left[(1-x^2)\dfrac{\partial v}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[y^2\dfrac{\partial v}{\partial y}\right]=0$

b) If $z=x^n f\left(\dfrac{y}{x}\right)+y^{-n}\phi\left(\dfrac{x}{y}\right)$ then, prove that

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=n^2 z$$

c) If $u=x\log xy$, where $x^3+y^3+3xy=1$ Find $\dfrac{du}{dx}$.

OR

Q10) Solve (Any two) **[16]**

a) Find the value of n, so that $u=r^n(3\cos^2\theta-1)$ satisfy the equation

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)=0$$

b) If $x^2 = au + bv$, $y = au - bv$ Prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$

c) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$ Find $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$

Q11) a) The area of a triangle ABC is calculated from the formula $\Delta = \frac{1}{2} bc \sin A$. Errors of 1%, 2%, and 3% respectively are made in measuring b, c, A if the correct value of A is 45°, find the % error in the calculated value of area of triangle. **[6]**

b) For the transformation $x = e^u \cos v$, $y = e^u \sin v$, Prove that **[6]**

$$\frac{\partial(x, y)}{\partial(u, v)} \cdot \frac{\partial(u, v)}{\partial(x, y)} = 1$$

c) Find the extreme values of $xy(a - x - y)$. **[6]**

OR

Q12) a) If $x + y + z = u$, $y + z = uv$, $z = uvw$, show that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$ **[6]**

b) Examine whether *u*, *v* are functionally dependent. If so find the relation between them $u = \frac{x - y}{y + x}$ $v = \frac{x + y}{x}$ **[6]**

c) Find the points on the surface $z^2 = xy + 1$ nearest to the origin, by using Lagrange's method. **[6]**

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