F.E. Sem – I
Nov – Dec – 2012
2008 course

Seat No.

[4261] – 101

F.E. Semester – I Examination, 2012
ENGINEERING MATHEMATICS – I
(2008 Pattern)

Time : 3 Hours   Max. Marks : 100

Instructions : 1) *Q.**1** or Q.**2**, Q.**3** or Q.**4**, Q.**5** or Q.**6**, from Section **I** and Q.**7** or Q.**8**, Q. **9** or Q.**10**, Q.**11** or Q.**12** from Section **II**.*
2) *Answers to the **two** Sections should be written in **separate** books.*
3) ***Neat** diagram must be drawn **wherever** necessary.*
4) *Figures to the **right** indicate **full** marks.*
5) ***Use** of logarithmic tables, slide rule, electronic pocket calculator is **allowed**.*
6) *Assume suitable data, **if necessary**.*

SECTION – I

1. A) Find the rank of matrix A by reducing it to its normal form. 6

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

B) Examine the consistency of the system and if consistent, solve it. 5

$4x - 2y + 6z = 8$

$x + y - 3z = -1$

$15x - 3y + 9z = 21$

C) Verify Cayley-Hamiltone theorem for the matrix A and find A^4. 7

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

OR

P.T.O.

2. A) Find eigen values and eigen vectors for the matrix A. **7**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

B) Examine whether the following vectors are linearly dependent. If so find the relation between them. **6**

$X_1 = [1 \; -1 \;\; 2 \;\; 2]$, $X_2 = [2 - 3 \;\; 4 - 1]$, $X_3 = [-1 \;\; 2 \; -2 \; 3]$

C) Given the transformation

$y_1 = 2x_1 + x_2 + x_3$

$y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

Find the co-ordinates (x_1, x_2, x_3) in X corresponding to (1, 2, – 1) in Y. **5**

3. A) Prove that : **6**

$$\left(1-e^{i\theta}\right)^{-1/2} + \left(1-e^{-i\theta}\right)^{-1/2} = \left(1+\operatorname{cosec}\tfrac{\theta}{2}\right)^{1/2}.$$

B) Find all the values of $(1+i)^{1/3}$ **5**

C) Prove that : **5**

$$\operatorname{Log}\tan\left(\frac{\pi}{4}+i\frac{x}{2}\right) = i\tan^{-1}(\sinh x)$$

OR

4\. A) A square lies above the real axis on an Argand's diagram and two of its adjacent vertices are the origin and the point $5 + 6i$. Find the complex numbers representing other vertices. 6

B) If $\alpha + i\beta = \tanh\left(x + i\frac{\pi}{4}\right)$, Prove that $\alpha^2 + \beta^2 = 1$. 5

C) Find the complex number Z so that, $\arg(Z+2) = \frac{\pi}{4}$ and $\arg(Z-2) = \frac{3\pi}{4}$. 5

5\. A) Find the n^{th} order differential co-efficient for $y = \frac{x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}$ 5

B) If $\sin^{-1} y = 2 \log (x + 1)$, then prove that

$(x + 1)^2 y_{n+2} + (2n + 1)(x+1) y_{n+1} + (n^2 + 4) y_n = 0.$ 5

C) Test the convergence of the series (any **one**) : 6

i) $\frac{2}{1} + \frac{2.5}{1.5} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + \ldots$

ii) $\frac{1}{1.2.3} + \frac{3}{2.3.4.} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \ldots$

OR

6\. A) Find the n^{th} order differential co-efficient for $y = \cos^4 x$. 5

B) If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ show that $(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$. 5