



**F.E. (Semester – I) Examination, 2011**  
**ENGINEERING MATHEMATICS – I**  
**(2008 Pattern)**

Time : 3 Hours

Max. Marks : 100

- Instructions :** 1) Answers to the **two** Sections should be written in **separate** books.  
 2) **Neat** diagrams must be drawn **wherever** necessary.  
 3) **Black** figures to the **right** indicate **full** marks.  
 4) Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is **allowed**.  
 5) Assume suitable data, **if** necessary.

**SECTION – I**

1. a) Define normal form of the matrix. Reduce the following matrix A to its normal form and hence find its rank. 6

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

- b) Examine the consistency of the system of the following equations. If consistent, solve system of equations : 6

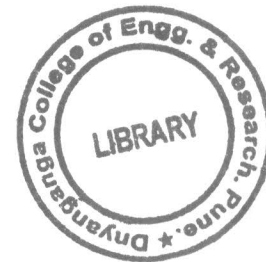
$$2x_1 - 3x_2 + 5x_3 = 1$$

$$3x_1 + x_2 - x_3 = 2$$

$$x_1 + 4x_2 - 6x_3 = 1$$

- c) Verify Cayley – Hamilton theorem for the matrix 6

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$



OR

P.T.O.



2. a) Find Eigen values and corresponding Eigen vectors for the matrix

7

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

- b) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

5

$$\bar{X}_1 = (3, 1, -4), \bar{X}_2 = (2, 2, -3), \bar{X}_3 = (0, -4, 1).$$

- c) Show that the transformation

$$y_1 = \frac{2}{3}x_1 + \frac{1}{3}x_2 + \frac{2}{3}x_3$$

$$y_2 = \frac{-2}{3}x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3$$

$$y_3 = \frac{1}{3}x_1 + \frac{2}{3}x_2 - \frac{2}{3}x_3$$

is orthogonal.

6

3. a) If  $|i + z| = |i - z|$ , show that  $z$  is real.

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- b) Find all the values of  $(1 + i)^{1/5}$ . Show that the continued product of these values is  $(1 + i)$ .

6

- c) If  $i^{(\alpha+i\beta)} = \alpha + i\beta$ , prove that  $\alpha^2 + \beta^2 = e^{-(4m+1)\pi\beta}$ .

5

OR

4. a) If  $z_1, z_2$  and  $z_3$  represent vertices of an equilateral triangle, prove that

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1.$$

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b) If  $\tan(\alpha + i\beta) = x + iy$ , prove that

$$x^2 + y^2 + 2x \cot 2\alpha = 1 \text{ and}$$

$$x^2 + y^2 - 2y \cot 2\beta + 1 = 0. \quad 6$$

c) If  $\alpha = 1 + i$ ,  $\beta = 1 - i$  and  $\cot \phi = x + 1$ ,

$$\text{prove that } \frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta} = \sin n\phi \operatorname{cosec}^n \phi. \quad 5$$

5. a) If  $y = e^x \sin 4x \cos 6x$

then find  $n^{\text{th}}$  order differential coefficient of  $y$  with respect to  $x$ . 5

b) If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$  then prove that

$$(1 - x^2) y_{n+2} - (2n + 3) x y_{n+1} - (n + 1)^2 y_n = 0. \quad 5$$

c) Test convergence of the series (**any one**): 6

i)  $\sum_{n=1}^{\infty} \frac{x^n}{x+n}$

ii)  $\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \dots$

**OR**

6. a) Find  $n^{\text{th}}$  order differential coefficient of  $y$  with respect to  $x$  if

$$y = \frac{x^2}{(x+2)(2x+3)}. \quad 5$$

b) If  $y = \cos(m \log x)$ , show that  $x^2 y_{n+2} + (2n+1) x y_{n+1} + (m^2 + n^2) y_n = 0$ . 5

