F.E. Sem-I Nov-Dec-2011

[4061] – 101

F.E. (Semester – I) Examination, 2011
ENGINEERING MATHEMATICS – I
(2008 Pattern)

Time : 3 Hours　　　　Max. Marks : 100

Instructions : 1) *Answers to the* ***two*** *Sections should be written in* ***separate*** *books.*
2) ***Neat*** *diagrams must be drawn* ***wherever*** *necessary.*
3) *Black figures to the* ***right*** *indicate* ***full*** *marks.*
4) *Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is* ***allowed.***
5) *Assume suitable data,* ***if necessary.***

SECTION – I

1. a) Define normal form of the matrix. Reduce the following matrix A to its normal form and hence find its rank. **6**

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

b) Examine the consistency of the system of the following equations. If consistent, solve system of equations : **6**

$2x_1 - 3x_2 + 5x_3 = 1$

$3x_1 + x_2 - x_3 = 2$

$x_1 + 4x_2 - 6x_3 = 1$

c) Verify Cayley – Hamilton theorem for the matrix **6**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

OR

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2. a) Find Eigen values and corresponding Eigen vectors for the matrix **7**

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

b) Examine whether the following vectors are linearly dependent. If so, find the relation between them : **5**

$\overline{X}_1 = (3, 1, -4), \overline{X}_2 = (2, 2, -3), \overline{X}_3 = (0, -4, 1).$

c) Show that the transformation

$$y_1 = \frac{2}{3}x_1 + \frac{1}{3}x_2 + \frac{2}{3}x_3$$

$$y_2 = \frac{-2}{3}x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3$$

$$y_3 = \frac{1}{3}x_1 + \frac{2}{3}x_2 - \frac{2}{3}x_3$$

is orthogonal. **6**

3. a) If $|i + z| = |i - z|$, show that z is real. **5**

b) Find all the values of $(1+i)^{1/5}$. Show that the continued product of these values is (1 + i). **6**

c) If $i^{(\alpha+i\beta)} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4m+1)\pi\beta}$. **5**

OR

4. a) If z_1, z_2 and z_3 represent vertices of an equilateral triangle, prove that

$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$. **5**

b) If $\tan(\alpha + i\beta) = x + iy$, prove that
$x^2 + y^2 + 2x \cot 2\alpha = 1$ and
$x^2 + y^2 - 2y \cot h2\beta + 1 = 0$. 6

c) If $\alpha = 1 + i$, $\beta = 1 - i$ and $\cot \phi = x + 1$,

prove that $\dfrac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \sin n\phi \operatorname{cosec}^n \phi$. 5

5. a) If $y = e^x \sin 4x \cos 6x$
then find n^{th} order differential coefficient of y with respect to x. 5

b) If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$ then prove that

$(1 - x^2)\, y_{n+2} - (2n + 3)\, xy_{n+1} - (n + 1)^2\, y_n = 0$. 5

c) Test convergence of the series (**any one**) : 6

i) $\sum_{n=1}^{\infty} \dfrac{x^n}{x+n}$

ii) $\sqrt{\dfrac{1}{2^3}} + \sqrt{\dfrac{2}{3^3}} + \sqrt{\dfrac{3}{4^3}} + \ldots$

OR

6. a) Find n^{th} order differential coefficient of y with respect to x if

$y = \dfrac{x^2}{(x+2)(2x+3)}$. 5

b) If $y = \cos(m \log x)$, show that $x^2 y_{n+2} + (2n + 1)\, xy_{n+1} + (m^2 + n^2)\, y_n = 0$. 5