F.E. Nov-Dec
Sem - I 2010

Total No. of Questions : 12] [Total No. of Printed Pages : 7

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F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) *Answers to the* ***two sections*** *should be written in* ***separate answer books****.*

(2) *Black figures to the right indicate full marks.*

(3) *Neat diagrams must be drawn wherever necessary.*

(4) *Assume suitable data, if necessary.*

(5) *Use of electronic pocket calculator is allowed.*

SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where **[05]**

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations : **[06]**

$x + y - z + t = 2$

$2x + 3y + 4t = 9$

$y - 2z + 3t = 2$

(C) Verify Cayley Hamilton Theorem for the matrix **[07]**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR

Q.2) (A) Find Eigen Values and corresponding Eigen Vectors for the matrix **[07]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(B) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

$X_1 = (2, -2, 4)$, $X_2 = (-1, 3, -3)$, $X_3 = (1, 1, 1)$ **[05]**

(C) Find values of a, b, c so that the matrix

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

becomes an orthogonal matrix. **[06]**

Q.3) (A) If $\dfrac{Z-1}{Z+i}$ is a purely imaginary number, then show that the locus of Z is a circle. **[06]**

(B) Show that the continued product of all values of $\left(1 + i\sqrt{3}\right)^{\frac{1}{4}}$ is $2\left[\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right]$. **[05]**

(C) If α, β are roots of an equation,

$\sin^2\theta \, z^2 - (2\sin\theta\cos\theta) \, z + 1 = 0$, prove that

$\alpha^n + \beta^n = 2\cos n\theta \, \mathrm{cosec}^n\theta$, where n is an integer. **[05]**

OR

Q.4) (A) Find tanhx if $5 \sinh x - \cosh x = 5$ **[05]**

(B) If $u + iv = \sin(x + iy)$,

prove that :

(a) $u^2\mathrm{cosec}^2x - v^2\sec^2x = 1$

(b) $u^2\mathrm{sech}^2x + v^2\mathrm{cosech}^2x = 1$ **[05]**

(C) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point 3 + 2i. Find the rest two vertices of the square. **[06]**

Q.5) (A) If $y = \dfrac{x^3}{x^2 - 1}$,

then find n^{th} order differential coefficient of y w.r. to x. **[05]**

(B) If $y = \sin^{-1}[3x - 4x^3]$,

prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, x y_{n+1} - n^2 y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $1 + \dfrac{1}{2} + \dfrac{1 \cdot 3}{2 \cdot 4} + \dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \ldots\ldots$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 1)\, x^n}{n^2}$

OR

Q.6) (A) If $y = (2x + 1) \log (4x + 3)$, then find y_{20}. **[05]**

(B) If $y = \left[x + \sqrt{x^2 - 1}\right]^m$,

prove that $(x^2 - 1) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - m^2) y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \ldots\ldots$

(b) $\sum_{n=1}^{\infty} \frac{4.7.10 \ldots\ldots (3n+1)}{1.2.3.4 \ldots\ldots n}$

SECTION - II

Q.7) (A) Expand $\frac{x}{e^x - 1}$ upto x^4. **[05]**

(B) Use Taylor's Theorem to obtain approximate value of $\sqrt{10}$ to four decimal places. **[05]**

(C) Solve : **(Any One)** **[06]**

(a) Find a and b, if

$$\lim_{x \to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = -\frac{1}{2}.$$

(b) Evaluate $\lim_{x \to 0} \left(\sin^2 \frac{\pi}{2 - ax}\right)^{\sec^2 \frac{\pi}{2-bx}}$

OR