

Total No. of Questions : 12]

[Total No. of Printed Pages : 7

[3461]-101

F. E. (2008 Course) Examination - 2008

ENGINEERING MATHEMATICS - I

Time : 3 Hours]

[Max. Marks : 100

Instructions :

- (1) In section I, attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6. In section II, attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.
- (2) Answers to the **two sections** should be written in **separate answer-books**.
- (3) Figures to the right indicate full marks.
- (4) Neat diagrams must be drawn wherever necessary.
- (5) Use of non-programmable electronic pocket calculator is allowed.
- (6) Assume suitable data, if necessary.

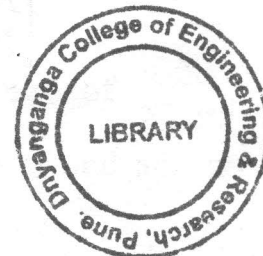
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SECTION - I

Q.1) (A) Define Rank of a matrix. Reduce the following matrix to the normal form and hence find its rank. [06]

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & -4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$$



$$x + y + z - w = -2$$

$$3x + 2y - z = 6$$

$$4y + 3z + 2w = -8$$

- (C) Verify Cayley-Hamilton theorem for the matrix : [05]

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Hence find  $A^{-1}$  if it exists

OR

- Q.2) (A) Find the Eigen values and corresponding Eigen vectors of the matrix : [06]

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

- (B) Examine for linear dependence the system of vectors  
 $x_1 = (2, -1, 3, 2)$ ;  $x_2 = (1, 3, 4, 2)$ ;  $x_3 = (3, -5, 2, 2)$  [06]

If dependent, find the relation between them.

- (C) If  $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & -\frac{2}{3} & c \end{bmatrix}$  is orthogonal, find a, b, c. [05]

Q.3) (A) If  $\arg(z + 1) = \frac{\pi}{6}$  and  $\arg(z - 1) = \frac{2\pi}{3}$ , find  $z$ . [05]

(B) If  $\frac{z - 2i}{2z - 1}$  is purely imaginary, prove that the locus of  $z$  in the Argand diagram is a circle. Find its center and radius. [06]

(C) If  $\cos(\theta + i\phi) = \cos\alpha + i\sin\alpha$ , prove that  $\sin^2\theta = \pm\sin\alpha$  [05]

OR

Q.4) (A) Prove that

$$\frac{|\operatorname{Re}(z)| + |\operatorname{Im}(z)|}{\sqrt{2}} \leq |z| \quad [05]$$

(B) Find all roots of the equation  $x^9 + x^5 - x^4 - 1 = 0$  [06]

(C) Find the principal value of  $i^{(1-i)}$  [05]

Q.5) (A) Test for convergence of the following series : (Any Two) [08]

(1)  $\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$

(2)  $\frac{2x}{1^2} + \frac{3^2x^2}{2^3} + \frac{4^3x^3}{3^4} + \frac{5^4x^4}{4^5} + \dots$  for  $x \neq 1$

(3)  $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots$

(B) Prove that the value of the  $n^{\text{th}}$  derivative of  $\frac{x^3}{x^2 - 1}$  for  $x = 0$  is zero if  $n$  is even and  $(-n!)$  if  $n$  is odd and greater than 1. [04]

- (C) If  $y = a \cos(\log x) + b \sin(\log x)$ , show that  
 $x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0$  [05]

OR

- Q.6 (A) Test for convergence of the following series. [08]

(1)  $\sum_{n=1}^{\infty} \frac{a^{n+1}}{n^n}$

(2)  $\sum_{n=1}^{\infty} \frac{1^2 5^2 9^2 \dots (4n-3)^2}{4^2 8^2 12^2 \dots (4n)^2}$

(3)  $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + n + 1} - \sqrt{n^2 - n + 1}}{n}$

- (B) Find  $n^{\text{th}}$  derivative of  $\frac{x^2}{(x+2)(2x+3)}$  [04]

- (C) If  $y = \log(x + \sqrt{1+x^2})$ , then prove that  
 $(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0$  [05]

## SECTION - II

- Q.7 (A) Attempt any two : [08]

- (1) Expand  $\log\left[\frac{xe^x}{e^x-1}\right]$  in ascending powers of  $x$  up to term in  $x^4$ .
- (2) Expand  $x^4 - 3x^3 + 2x^2 - x + 1$  in powers of  $(x-3)$ .
- (3) Using Taylor's Theorem, find  $\sin(30^\circ 30')$

