

Total No. of Questions : 12]

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[3461]-101

F. E. (2008 Course) Examination - 2008

ENGINEERING MATHEMATICS - I

Time : 3 Hours]

[Max. Marks : 100

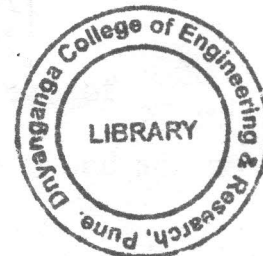
Instructions :

- (1) In section I, attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6. In section II, attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.
- (2) Answers to the **two sections** should be written in **separate answer-books**.
- (3) Figures to the right indicate full marks.
- (4) Neat diagrams must be drawn wherever necessary.
- (5) Use of non-programmable electronic pocket calculator is allowed.
- (6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Define Rank of a matrix. Reduce the following matrix to the normal form and hence find its rank. [06]

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & -4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$$



$$x + y + z - w = -2$$

$$3x + 2y - z = 6$$

$$4y + 3z + 2w = -8$$

- (C) Verify Cayley-Hamilton theorem for the matrix : [05]

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Hence find A^{-1} if it exists

OR

- Q.2) (A) Find the Eigen values and corresponding Eigen vectors of the matrix : [06]

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

- (B) Examine for linear dependence the system of vectors
 $x_1 = (2, -1, 3, 2)$; $x_2 = (1, 3, 4, 2)$; $x_3 = (3, -5, 2, 2)$ [06]

If dependent, find the relation between them.

- (C) If $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & -\frac{2}{3} & c \end{bmatrix}$ is orthogonal, find a, b, c. [05]

Q.3) (A) If $\arg(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$, find z . [05]

(B) If $\frac{z - 2i}{2z - 1}$ is purely imaginary, prove that the locus of z in the Argand diagram is a circle. Find its center and radius. [06]

(C) If $\cos(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that $\sin^2\theta = \pm\sin\alpha$ [05]

OR

Q.4) (A) Prove that

$$\frac{|\operatorname{Re}(z)| + |\operatorname{Im}(z)|}{\sqrt{2}} \leq |z| \quad [05]$$

(B) Find all roots of the equation $x^9 + x^5 - x^4 - 1 = 0$ [06]

(C) Find the principal value of $i^{(1-i)}$ [05]

Q.5) (A) Test for convergence of the following series : (Any Two) [08]

(1) $\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$

(2) $\frac{2x}{1^2} + \frac{3^2x^2}{2^3} + \frac{4^3x^3}{3^4} + \frac{5^4x^4}{4^5} + \dots$ for $x \neq 1$

(3) $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots$

(B) Prove that the value of the n^{th} derivative of $\frac{x^3}{x^2 - 1}$ for $x = 0$ is zero if n is even and $(-n!)$ if n is odd and greater than 1. [04]

- (C) If $y = a \cos(\log x) + b \sin(\log x)$, show that
 $x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0$ [05]

OR

- Q.6 (A) Test for convergence of the following series. [08]

(1) $\sum_{n=1}^{\infty} \frac{a^{n+1}}{n^n}$

(2) $\sum_{n=1}^{\infty} \frac{1^2 5^2 9^2 \dots (4n-3)^2}{4^2 8^2 12^2 \dots (4n)^2}$

(3) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + n + 1} - \sqrt{n^2 - n + 1}}{n}$

- (B) Find n^{th} derivative of $\frac{x^2}{(x+2)(2x+3)}$ [04]

- (C) If $y = \log(x + \sqrt{1+x^2})$, then prove that
 $(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0$ [05]

SECTION - II

- Q.7 (A) Attempt any two : [08]

- (1) Expand $\log\left[\frac{xe^x}{e^x-1}\right]$ in ascending powers of x up to term in x^4 .
- (2) Expand $x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$.
- (3) Using Taylor's Theorem, find $\sin(30^\circ 30')$

