Total No. of Questions—**12**] [Total No. of Printed Pages—**7**

Seat No.	

[4756]-11

F.E. (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS–I

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data if necessary.

SECTION I

1. (A) Reduce the following matrix A to its normal form and hence find its rank, where [5]

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}.$$

(B) Is the following system of equations consistent ? If so solve it : [6]

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

$$2x - 2y + 3z = 7$$

(C) Verify Cayley-Hamilton theorem for the matrix : [7]

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

Or

2. (A) Find Eigenvalues and corresponding Eigenvectors for the matrix : [7]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

(B) Examine whether the following vectors are linearly dependent. If so find the relation between them : [5]

$$\bar{X}_1 = (3, 1, -4), \bar{X}_2 = (2, 2, -3), \bar{X}_3 = (0, -4, 1).$$

(C) Show that :

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. [6]

3. (A) If Z_1 and Z_2 are two complex numbers such that :

$|Z_1 + Z_2| = |Z_1 - Z_2|$, then show that $\operatorname{amp}\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{2}$. [6]

(B) Find the continued product of the four values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{1/4}$.

[5]

(C) If $p \log(a + ib) = (x + iy) \log m$, prove that : [5]

$$\frac{y}{x} = \frac{2\tan^{-1}\frac{b}{a}}{\log(a^2 + b^2)}.$$

Or

4. (A) If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, prove that : [5]

(*i*) $\tanh\frac{y}{2} = \tan\frac{x}{2}$

(*ii*) $\cosh y \cos x = 1.$

(B) If $\sin(\alpha + i\beta) = x + iy$, prove that : [5]

(*i*) $\dfrac{x^2}{\cosh^2 \beta} + \dfrac{y^2}{\sinh^2 \beta} = 1$

(*ii*) $\dfrac{x^2}{\sin^2 \alpha} - \dfrac{y^2}{\cos^2 \alpha} = 1$.

(C) A square lies above the real axis in Argand diagram, and two of its adjacent vertices are origin and the point $5 + 6i$. Find the complex numbers representing other vertices. [6]

5. (A) Find nth derivative of $y = x^2 e^x \cos x$. [5]

(B) If $y = A \cos(\log x) + B \sin(\log x)$ then show that $x^2 y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 + 1)y_n = 0.$ [5]

(C) Test convergence of the series (any *one*) : [6]

(*i*) $\displaystyle\sum_{n=1}^{\infty} \frac{2n + 1}{n^3 + 1} x^n$

(*ii*) $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{4}} \ldots\ldots$

Or

6. (A) If $y = \sin^{-1}(3x - 4x^3)$, prove that : [5]

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$$

(B) If

$$y = \frac{x}{(x-1)(x-2)(x-3)}$$

find nth order differential coefficient of y w.r.t. x. [5]

(C) Test convergence of the series (any *one*) : [6]

(*i*) $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} \ldots\ldots + \frac{n+1}{n^3} \ldots\ldots$

(*ii*) $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \ldots.$

SECTION II

7. (A) Expand $\sqrt{1 + \sin x}$ upto x^6. [5]

(B) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. [5]

(C) Solve (any *one*) : [6]

(*a*) If $\lim_{x \to 0} \frac{\sin 2x + p \sin x}{x^3}$ is finite, then find the value of p and hence the value of the limit.

(*b*) Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{x}\right)^{2 \sin x}.$$

Or

8. (A) Expand $\tan^{-1} x$ in ascending powers of x. [5]

(B) Using Taylor's theorem, express $(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in powers of x. [5]

(C) Solve (any *one*) : [6]

(*a*) Evaluate

$$\lim_{x \to 1}\left[\frac{x}{x-1} - \frac{1}{\log x}\right].$$

(*b*) Evaluate

$$\lim_{x \to 0}\frac{1 - x^x}{x \log x}.$$

9. Solve (any *two*) : [16]

(A) If $u = \log (x^3 + y^3 - x^2y - xy^2)$, prove that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}.$$

(B) If

$$x = \frac{x}{2}\left(e^\theta + e^{-\theta}\right),\ y = \frac{r}{2}\left(e^\theta - e^{-\theta}\right),$$

then show that :

$$\left(\frac{\partial x}{\partial r}\right)_\theta = \left(\frac{\partial r}{\partial x}\right)_y.$$

(C) Verify Euler's theorem for homogeneous function $u = \sqrt{x} + \sqrt{y} + \sqrt{z}$.

Or

10. Solve (any *two*) : [16]

(A) If

$$V = \frac{C}{\sqrt{t}} e^{-x^2/4a^2t}$$

then show that :

$$\frac{\partial V}{\partial t} = a^2 \frac{\partial^2 V}{\partial x^2}.$$

(B) If

$$u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right),$$

show that :

$$2x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = \tan u.$$

(C) If

$$u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2),$$

prove that :

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0.$$

11. (A) Find the percentage error in the area of an ellipse when an error of 1% is made in measuring its major and minor axis. [6]

(B) If $x + y + z = u$, $y + z = uv$, $z = uvw$, find [6]

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}.$$

(C) Determine the points where the function $x^3 + y^3 - 3axy$ has maximum or minimum values. [6]

Or

12. (A) Verify $JJ' = 1$ for $x = e^u \cos v$, $y = e^u \sin v$. [6]

(B) Examine for functional dependence/independence. If dependent, find relation between them : [6]

$$u = \frac{x + y}{1 - xy}, \quad v = \tan^{-1} x + \tan^{-1} y.$$

(C) Use Lagrange's method to find the minimum distance from origin to the plane $3x + 2y + z = 12$. [6]