

Total No. of Questions : 12]

SEAT No. :

[Total No. of Pages : 5

P2022

F.E.
ENGINEERING MATHEMATICS - I
(2008 Pattern)

Time : 3 Hours]

[Max. Marks :100

Instructions to the candidates:

- 1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6 from section - I and Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12 from section - II.*
- 2) *Answers to the two sections should be written in separate answer books.*
- 3) *Figures to the right indicate full marks.*
- 4) *Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.*
- 5) *Assume suitable data, if necessary.*

SECTION - I

Q1) a) Reduce the following matrix to normal form and hence find its rank.

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix} \quad [5]$$

b) Find for what value of k, the set of equations [6]

$$2x - 3y + 6z - 5t = 3$$

$$y - 4z + t = 1$$

$$4x - 5y + 8z - 9t = k$$

has (i) No solution (ii) An infinite number of solutions.

c) Verify Caley - Hamilton theorem for the following matrix and use it to find A^{-1} . [7]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

OR

P.T.O.

- Q2) a)** Find Eigen values and the corresponding Eigenvectors of the following matrix. [6]

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

- b) Examine whether the following vectors are linearly dependent. If so find the relation between them: [6]

$$\bar{x}_1 = (3, 1, -4) \quad \bar{x}_2 = (2, 2, -3) \quad \bar{x}_3 = (0, -4, 1)$$

- c) For the given transformation [6]

$$Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

find the coordinates (x_1, x_2, x_3) corresponding to $(2, 9, 5)$ in Y.

- Q3) a)** Find all values of $(1 + i)^{1/5}$. Show that their product is $1 + i$. [5]

- b) Find z if $\arg(z + 2i) = \pi/4$, $\arg(z - 2i) = 3\pi/4$. [5]

- c) If $\tan(\alpha + i\beta) = x + iy$ prove that [6]

$$x^2 + y^2 + 2x \cot 2\alpha = 1 \text{ and } x^2 + y^2 - 2y \coth \beta = -1$$

OR

- Q4) a)** Solve the equation $x^9 - x^5 + x^4 - 1 = 0$ using De Moivre's theorem. [5]

- b) A square lies above real axis in Argand diagram, and two of its adjacent vertices are the origin & the point $5 + 6i$. Find the complex numbers representing other two vertices of the square. [6]

- c) Prove that $\tan \left\{ i \log \left(\frac{a - ib}{a + ib} \right) \right\} = \frac{2ab}{a^2 + b^2}$. [5]

- Q5) a)** Find n^{th} derivative of $y = e^x \cos x \cos 2x$. [5]

- b) If $y = e^{m \cos^{-1} x}$ prove that [5]

$$(1 - x^2) y_{n+2} - (2n + 1)x y_{n+1} - (n^2 + m^2)y_n = 0$$

- c) Discuss convergence of the following series (any one) : [6]

i) $\frac{1}{1+z} + \frac{2}{1+z^2} + \frac{3}{1+z^3} + \dots$

ii) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n^3} \right) x^n, x > 0$

OR

Q6) a) Find n^{th} derivative of $y = \frac{x}{(x-1)(x-2)(x-3)}$. [5]

b) If $y = (x + \sqrt{x^2 - 1})^m$ show that

$$(x^2 - 1)y_{n+2} + (2n + 1)x y_{n+1} + (n^2 - m^2)y_n = 0 \quad [5]$$

c) Discuss convergence of the following series (any one) : [6]

i) $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots$

ii) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

SECTION - II

Q7) a) Expand $\log \cos \left(x + \frac{\pi}{4} \right)$ using Taylor's theorem in ascending power of x up to x^3 . [5]

b) Obtain the Maclaurin's expansion of $\tan \left(\frac{\pi}{4} + x \right)$ and hence find the value of $\tan(46.5^\circ)$ to three places of decimals. [5]

c) Solve (any one)

i) Find a and b if

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$$

ii) Prove that [6]

$$\lim_{x \rightarrow \infty} \left(\frac{ax+1}{ax-1} \right)^x = e^{2/a}$$

OR

Q8) a) Using Taylor's theorem express [5]

$7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4$ in ascending powers of x .

b) Obtain the expansion of $\sinh^{-1}(3x + 4x^3)$ upto a term in x^5 . [5]

c) Solve (any one) [6]

i)
$$\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}.$$

ii)
$$\lim_{x \rightarrow \infty} \left\{ x - x^2 \log \left(1 + \frac{1}{x} \right) \right\}.$$

Q9) Solve : (Any two) [16]

a) If $v = (1 - 2xy + y^2)^{-1/2}$ prove that

i)
$$x \frac{\partial v}{\partial x} - y \frac{\partial v}{\partial y} = y^2 v^3$$

ii)
$$\frac{\partial}{\partial x} \left[(1 - x^2) \frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial y} \left[y^2 \frac{\partial v}{\partial y} \right] = 0.$$

b) If $x = u \tan v$, $y = u \sec v$

prove that

$$\left(\frac{\partial u}{\partial x} \right)_y \left(\frac{\partial v}{\partial x} \right)_y = \left(\frac{\partial u}{\partial y} \right)_x \left(\frac{\partial v}{\partial y} \right)_x$$

c) If $u = \sin^{-1} \sqrt{x^2 + y^2}$ prove that

i)
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$$

ii)
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u$$

OR

Q10) Solve : (Any two)

[16]

a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x + y + z)^2}$$

b) If $u = \operatorname{cosec}^{-1} \sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$ then, show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left[\frac{13}{12} + \frac{\tan^2 u}{12} \right]$$

c) If $u = x \log xy$ where

$$x^3 + y^3 + 3xy = 1 \text{ find } \frac{du}{dx}.$$

Q11)a) Show that $JJ' = 1$ for

[6]

$$x = e^v \sec u \text{ and } y = e^v \tan u$$

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively. Find the error in the calculated volume. [6]

c) Determine the points where the function $x^3 + y^3 - 3axy$ has a maximum or minimum. [6]

OR

Q12)a) If $x = a(u + v)$, $y = b(u - v)$ and $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$ using chain

rule find $\frac{\partial(x, y)}{\partial(r, \theta)}$. [6]

b) Examine whether u, v are functionally dependent. If so find the relation between them. [6]

$$u = \frac{x + y}{1 - xy}, \quad v = \tan^{-1} x + \tan^{-1} y$$

c) If $u = xyz$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$ then find $\frac{\partial x}{\partial u}$. [6]

