

F.E. sem - I

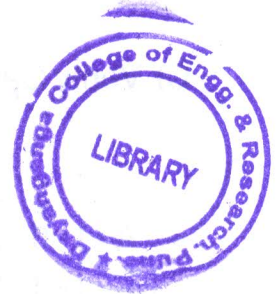
May-June-2012 [4161] - 101



Seat
No.

--

F.E. Semester – I Examination, 2012
ENGINEERING MATHEMATICS – I
(2008 Pattern)



Time : 3 Hours

Max. Marks : 100

- Instructions :**
- I) Attempt Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6 from Section I and Q. 7 or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12 from Section II.
 - II) Answers to the **two** Sections should be written in **separate** books.
 - III) Neat diagrams must be drawn wherever necessary.
 - IV) Black figures to the right indicate full marks.
 - V) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.
 - VI) Assume suitable data, if necessary.

SECTION – I

1. A) Define Rank of the matrix. Find the rank of matrix A by reducing it to its normal form. 6

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

- B) Show that the system 5

$$3x + 4y + 5z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$5x + 6y + 7z = \gamma$$

is consistent only when α, β, γ are in geometric progression.

P.T.O.



C) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix} \text{ and hence find } A^{-1}.$$

7

OR

2. A) Find Eigen values and Eigen vectors for the matrix

7

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B) Examine whether the following vectors are linearly dependent. If so find the relation between them $x_1 = (1, 1, 1, 3)$, $x_2 = (1, 2, 3, 4)$, $x_3 = (2, 3, 4, 7)$.

6

C) Given the transformation

$$Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the co-ordinates (x_1, x_2, x_3) corresponding to $(2, 9, 5)$ in Y.

5

3. A) Prove that

$$\frac{1 + \cos \alpha + i \sin \alpha}{1 - \cos \alpha + i \sin \alpha} = \left(\cot \frac{\alpha}{2} \right) \left[e^{i \left(\frac{\alpha - \pi}{2} \right)} \right].$$

5

B) Solve the equation $x^7 - x^4 + x^3 - 1 = 0$ by using De Moivre's theorem.

5



C) Prove that

$$\log\left[\frac{1}{1-e^{i\theta}}\right] = \log\left(\frac{1}{2}\operatorname{cosec}\frac{\theta}{2}\right) + i\left(\frac{\pi}{2} - \frac{\theta}{2}\right). \quad 6$$

OR

4. A) If z_1, z_2 and origin represent vertices of an equilateral triangle on the Argand diagram, show that

6

$$\frac{1}{z_1^2} + \frac{1}{z_2^2} = \frac{1}{z_1 z_2}$$

B) If $\operatorname{cosec}\left(\frac{\pi}{4} + ix\right) = u + iv$ then prove that

$$(u^2 + v^2)^2 = 2(u^2 - v^2).$$

5

C) If $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma}$ then prove that

$$\frac{(a+b)(b+c)(c+a)}{abc} = 8\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\beta-\gamma}{2}\right)\cos\left(\frac{\gamma-\alpha}{2}\right).$$

5

5. A) If $y = \frac{1}{(x-2)(x-1)^2}$

then find n^{th} order differential coefficient of y w.r.t. x .

5

B) If $x = \tan(\log y)$ prove that,

$$(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + [n^2-n-2]y_n = 0.$$

5

