May - June 2010
Sem-I
F.E. (2008 Course)

Total No. of Questions : 12] [Total No. of Printed Pages : 7

[3761]-101

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) *Solve Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.*

(2) *Answers to the* ***two sections*** *should be written in* ***separate answer-books.***

(3) *Figures to the right indicate full marks.*

(4) *Neat diagrams must be drawn wherever necessary.*

(5) *Use of non-programmable calculator is allowed.*

(6) *Assume suitable data, if necessary.*

SECTION - I

Q.1) (A) Define Rank of a Matrix. Reduce the following matrix to the normal form and hence find its rank : **[06]**

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

(B) Examine consistency of the following system of equations : **[06]**

$2x - y - z = 2,\ x + 2y + z = 2,\ 4x - 7y - 5z = 2.$

(C) Verify Cayley Hamilton Theorem for : **[05]**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

Hence find A^{-1}.

OR

Q.2) (A) Find eigen values and eigen vectors for the following matrix : **[07]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(B) Examine whether following vectors are linearly dependent. If so find the relation amongst them : **[05]**

$X_1 = (3, 1, -4)$; $X_2 = (2, 2, -3)$; $X_3 = (0, -4, 1)$

(C) Show that the transformation : **[05]**

$y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$

is non-singular. Also find the values of x_1, x_2, x_3 if

$y_1 = 1$, $y_2 = 2$, $y_3 = -1$ by using inverse transformation.

Q.3) (A) Find the complex no. z if $\arg(z + 2) = \frac{\pi}{4}$ and

$\arg(z - 2) = \frac{3\pi}{4}$. **[05]**

(B) Prove that $\left(\frac{-1 + i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n = -1$ if $n = 8$

$= 2$ if $n = 9$. **[05]**

(C) Find all fifth roots of unity. Show that all these roots form a geometric progression and also show that continued product of all 5^{th} roots is one. **[06]**

OR

Q.4) (A) Find a and b if $\cos^{-1}\left(\frac{3i}{4}\right) = a + ib$. **[05]**

(B) If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, prove that :

(1) $\tanh \frac{y}{z} = \tan x/2$

(2) $\cosh y . \cos x = 1$ **[05]**

(C) Two opposite vertices of a square are represented by complex nos. $(9 + 12i)$ and $(-5 + 10i)$. Find the complex no. representing the other two vertices of the square. **[06]**

Q.5) (A) Test the following series for convergence : **(Any Two)** **[08]**

(1) $\frac{1}{1\cdot 3} + \frac{2}{3\cdot 5} + \frac{3}{5\cdot 7} + \frac{4}{7\cdot 9} + \ldots\ldots\ldots$

(2) $\sum \frac{2^n}{n^4 + 1} x^n \;, \; x > 0$

(3) $\frac{1}{2\cdot 4} + \frac{1\cdot 3}{2\cdot 4\cdot 6} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8} + \ldots\ldots\ldots$

(B) Find n^{th} derivative of $y = \tan^{-1} x$. **[04]**

(C) If $y = a \cos(\log x) + b \sin(\log x)$, show that

$x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0.$ **[05]**

OR

Q.6) (A) Test the following series for convergence : **(Any Two)** **[08]**

(1) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots \quad , x > 0$

(2) $1 + \frac{3}{2!} + \frac{3^2}{3!} + \frac{3^3}{4!} + \frac{3^4}{5!} + \ldots\ldots\ldots$

(3) $1 + \frac{3}{7} + \frac{3\cdot 6}{7\cdot 10} + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13} + \frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16} + \ldots\ldots\ldots$

(B) Find the n^{th} derivative of $y = x^2 e^{3x} \sin 4x$ **[04]**

(C) If $y = (\sin^{-1} x)^2$, find the relation between y_{n+2}, y_{n+1} and y_n. **[05]**

SECTION - II

Q.7) (A) Solve **any two** : **[08]**

(1) Expand six x.cosh x in ascending powers of x upto term in x^5.

(2) Use Taylor's Theorem to find $\sqrt{25.15}$.

(3) Expand $\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ in ascending powers of x upto term in x^7.

(B) Solve **any two** : **[08]**

(1) Evaluate $\lim_{x\to 0} \frac{\log(\tan x)}{\log x}$

(2) Evaluate $\lim_{x\to 0} x \log x$

(3) Evaluate $\lim_{x\to \infty} \left(\frac{1}{x}\right)^{1/x}$

OR