FE. Sem - I May - June 2009

Total No. of Questions : 12] [Total No. of Printed Pages : 7

[3561]-101

F. E. (Semester - I) Examination - 2009

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) Answer ***three*** *question from section I and* ***three*** *questions from section II.*

(2) Answer to the ***two sections*** *should be written in* ***separate answer-books.***

(3) Figures to the right indicate full marks.

(4) Neat diagrams must be drawn wherever necessary.

(5) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Find non-singular matrices P and Q such that PAQ is in normal form. Hence find the rank of A where

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix}$$ [06]

(B) Determine the values of λ for which the equations

$x + 2y + z = 3$

$x + y + z = \lambda$

$3x + y + 3z = \lambda^2$

are consistent and solve them for these values of λ. [06]

(C) Verify Cayley-Hamilton Theorem for

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ [05]

OR

Q.2) (A) Find the eigen values and corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$ [06]

(B) Examine for linear dependence the system of vectors [1, 2, –1, 0], [1, 3, 1, 2], [4, 2, 1, 0], [6, 1, 0, 1] and if dependent find the relation between them. [06]

(C) Determine the values of a, b, c when

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal. [05]

Q.3) (A) Show that $\left|\frac{z}{|z|} - 1\right| \leq |\arg z|$ [05]

(B) Find all the values of $(1+i)^{1/5}$ and show that their product is $(1 + i)$. [06]

(C) Express $\cos^{-1}\left(\frac{3i}{4}\right)$ in the form a + ib. **[05]**

OR

Q.4) (A) Prove that

$$\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2-b^2}.$$ **[05]**

(B) A square lies above real axis on Argand diagram and two of its adjacent vertices are origin and the point 5 + 6i. Find the complex numbers representing other vertices. **[06]**

(C) Prove that i^i is wholly real and find its principal value. Also show that the values of i^i form a geometric progression. **[05]**

Q.5) (A) Test for convergence of the following series : **(Any Two)** **[08]**

(1) $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\ldots+\frac{n^2}{n!}+\ldots$

(2) $\frac{2}{1}+\frac{3}{8}+\frac{4}{27}+\ldots+\frac{n+1}{n^3}+\ldots$

(3) $1+\frac{3}{4}x+\frac{5}{9}x^3+\frac{9}{65}x^4+\ldots$

(B) Using Leibnitz's theorem find n^{th} derivative of $x^2 e^{3x} \sin 4x$. **[04]**

(C) If $x = \sin\theta$, $y = \sin 2\theta$

prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0$$ **[05]**

OR

Q.6) (A) Test for convergence of the following series : **(Any Two)** [08]

(1) $\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \ldots$

(2) $\sum \frac{1^2 \cdot 5^2 \cdot 9^2 \ldots. (4n-3)^2}{4^2 \cdot 8^2 \cdot 12^2 \ldots (4n)^2}$

(3) $\sum \frac{x^n}{a + \sqrt{n}}$

(B) Find the n^{th} derivative of $\frac{x^4}{(x-1)(x-2)}$ [04]

(C) If $x = \tan(\log y)$ prove that

$(1 + x^2)\, y_{n+1} + (2nx - 1)\, y_n + n(n-1)\, y_{n-1} = 0$ [05]

SECTION - II

Q.7) (A) Expand $\cos^{-1}\left[\frac{x - x^{-1}}{x + x^{-1}}\right]$ in ascending powers of x. [04]

(B) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. [05]

(C) Attempt **any two** of the following : [08]

(1) Evaluate $\lim_{x \to 0} \frac{e^{2x} - (1+x)^2}{x \log (1+x)}$

(2) If $\lim_{x \to 0} \frac{\sin 2x + p \sin x}{x^3}$ is finite then find the value of p and hence the value of the limit.

(3) Evaluate $\lim_{x \to 1} \left[\frac{x}{x-1} - \frac{1}{\log x}\right]$

OR