Total No. of Questions—**12**] [Total No. of Printed Pages—**8+3**

| Seat No. | |
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# [4657]-31

**S.E. (Electrical/Inst./Comp./I.T.) (First Semester)**

**EXAMINATION, 2014**

**ENGINEERING MATHEMATICS—III**

**(2008 PATTERN)**

**Time : Three Hours** **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*v*) Neat diagrams must be drawn wherever necessary.

(*vi*) Assume suitable data, if necessary.

## SECTION I

**1.** (*a*) Solve any *three* : [12]

(*i*) $(D^2 + 4)\, y = x \sin x$

(*ii*) $\frac{d^2y}{dx^2} - y = x \sin x + \left(1 + x^2\right)e^x$

(*iii*) $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2\sin\left[\log(1 + x)\right]$

(*iv*) $\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$.

(*b*) A circuit consists of an inductance L and condenser of capacity C in series. An alternating e.m.f. E sin $nt$ is applied to it at time $t = 0$; the initial current and charge on the condenser being zero. Find the charge at any time for $w \neq n$; $w^2 = \frac{1}{\text{LC}}$. [5]

*Or*

**2.** (*a*) Solve any *three* : [12]

(*i*) $(\text{D}^2 + 2\text{D} + 1)\, y = xe^{-x} \cos x$

(*ii*) $\frac{d^3y}{dx^3} - y = \left(1 + e^x\right)^2$

(*iii*) $(\text{D}^2 + 1)\, y = \text{cosec}\, x$ [Use method of variation of parameter]

(*iv*) $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$.

(*b*) Solve : [5]

$$\frac{dx}{dt} + 2x - 3y = t$$

$$\frac{dy}{dt} + 2y - 3x = e^{2t}.$$

**3.** (*a*) If

$$u = \frac{1}{2}\log\left(x^2 + y^2\right) \text{ and}$$

$$f(z) = u + iv$$

is analytic then find $f(z)$ in terms of $z$ and hence find $v$. [5]

(*b*) Find bilinear transformation which maps the points $z = 1$, $i$, $-1$ to the points 0, 1, $\infty$ respectively. [6]

(*c*) By using Cauchy's formula evaluate : [5]

$$\oint \frac{e^z}{z^2 + 1}\, dz$$

over $|z - 1| = 1$.

*Or*

**4.** (*a*) Apply residue theorem to evaluate :

$$\oint_C \frac{z + 2}{z^2 + 1}\, dz$$

where C is $|z - i| = \frac{1}{2}$. [6]

(*b*) If both $f(z)$ and $\overline{f(z)}$ are analytic functions of $z$ then prove that $f(z)$ is constant function. [5]

(*c*) Find the map of straight line $y = x$ under the transformation : [5]

$$w = \frac{z - 1}{z + 1}.$$

**5.** (*a*) Find Fourier cosine transform of [5]

$$f(x) = \begin{cases} \cos x \,, & 0 < x < a \\ 0 \,, & x \geq a \end{cases}.$$

(*b*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \sin \lambda x \, dx = e^{-\lambda}, \qquad \lambda > 0.$$

(*c*) Find the $z$-transform of (any *two*) : [6]

(*i*) $f(k) = k5^k, \ (k \geq 0)$

(*ii*) $f(k) = e^{-3k} \sin 4k, \ (k \geq 0)$

(*iii*) $f(k) = \dfrac{e^{-k} - e^{-2k}}{k}.$

*Or*

**6.** (*a*) Use $z$-transform to solve : [5]

$$f(k+1) - f(k) = 1;$$

$$f(0) = 0;\ k \geq 0.$$

(*b*) Find inverse $z$-transform of (any *two*) : [6]

(*i*) $\dfrac{z}{(z-1)(z-2)};\ |z| > 2$

(*ii*) $\dfrac{z(z+1)}{(z-1)^2};\ |z| > 1$

(*iii*) $\dfrac{z}{(z-2)(z+4)^2}$

(Use inversion integral method.)

(*c*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \cos \lambda x \; dx = \begin{cases} 1 \; ; \; 0 < \lambda < 1 \\ 2 \; ; \; 1 < \lambda < 2 \\ 0 \; ; \quad \lambda > 2 \end{cases}.$$

## SECTION II

**7.** (*a*) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [9]

(*b*) The following are the values of import of raw material and export of finished product in suitable units :

| **Export** | **Import** |
|---|---|
| 10 | 12 |
| 11 | 14 |
| 14 | 15 |
| 14 | 16 |
| 20 | 21 |
| 22 | 26 |
| 16 | 21 |
| 12 | 15 |
| 15 | 16 |
| 13 | 14 |

Calculate the coefficient of correlation between the import values and export values. [8]

*Or*

**8.** (*a*) A can hit the target 1 out of 4 times, B can hit the target 2 out of 3 times, C can hit the target 3 out of 4 times. Find the probability of at least two hit the target. [6]

(*b*) Probability of a man aged 60 years will live for 70 years is $\frac{1}{10}$. Find the probability of 5 men selected at random 2 will live for 70 years. [5]

(*c*) Assuming that the diameter of 1000 brass plugs taken consecutively from machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many of the plugs are likely to be approved if the acceptable diameter is $0.752 \pm 0.004$.

[Area corresponding to 2.25 is 0.4878 and Area corresponding to 1.75 is 0.4599]. [6]

**9.** (*a*) The position vector of a particle at time $t$ is :

$$\bar{r} = \cos(t-1)\,\bar{i} + \sinh(t-1)\,\bar{j} + mt^3\,\bar{k}.$$

Find the condition imposed on $m$ by requiring that at time $t = 1$, the acceleration is normal to the position vector. [5]

(*b*) Find the directional derivative of the function :

$$\phi = e^{2x-y-z}$$

at (1, 1, 1) in the direction of the tangent to the curve

$$x = e^{-t},\ y = 2 \sin t + 1,\ z = t - \cos t$$

at $t = 0$. [5]

(*c*) Solve any *two* : [6]

(*i*) Show that the vector field $f(r)\bar{r}$ is always irrotational.

(*ii*) Show that :

$$\bar{a} \cdot \nabla\left[\bar{b} \cdot \nabla\left(\frac{1}{r}\right)\right] = \frac{3(\bar{a} \cdot \bar{r})(\bar{b} \cdot \bar{r})}{r^5} - \frac{(\bar{a} \cdot \bar{b})}{r^3}.$$

(*iii*) If $\bar{u}$ and $\bar{v}$ are irrotational vectors then prove that $\bar{u} \times \bar{v}$ is solenoidal vector.

*Or*

**10.** (*a*) Show that : [5]

$$\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$$

is irrotational. Find scalar $\phi$ such that :

$$\bar{F} = \nabla\phi.$$

(*b*) If the directional derivative of

$$\phi = axy + byz + cxz$$

at (1, 1, 1) has maximum magnitude 4 in a direction parallel to $x$-axis. Find the values of $a$, $b$, $c$. [5]

(*c*) Solve any *two* : [6]

(*i*) Show that :

$$\nabla^4 \left(r^2 \log r\right) = \frac{6}{r^2}.$$

(*ii*) Show that :

$$\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$$

is solenoidal field.

(*iii*) For constant vector $\bar{a}$, show that :

$$\nabla \times \left(\bar{a} \times \bar{r}\right) = 2\bar{a}.$$

**11.** (*a*) Evaluate : [5]

$$\oint_C x^2 \, dx + xy \, dy$$

over the curve 'C' bounded by $y = x^2$ and the line $y = x$ by using Green's theorem.

(*b*) Evaluate : [6]

$$\iint_S \overline{F} \cdot \overline{ds},$$

where 'S' is the surface of sphere

$$x^2 + y^2 + z^2 = 1$$

which lies in positive octant and

$$\overline{F} = yz\overline{i} + xz\overline{j} + xy\overline{k}.$$

(*c*) Evaluate : [6]

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot \overline{ds}$$

where

$$\overline{F} = \left(x^3 - y^3\right)\overline{i} - xyz\overline{j} + y^3\overline{k}$$

and 'S' is the surface

$$x^2 + 4y^2 + z^2 - 2x = 4$$

above the plane $x = 0$.

*Or*

**12.** (*a*) Find the work done in moving a particle once round the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1, \; z = 0$$

under the field of force given by : [5]

$$\overline{F} = (2x - y + z)\overline{i} + \left(x + y - z^2\right)\overline{j} + (3x - 2y + 4z)\overline{k}.$$

(*b*) Evaluate : [6]

$$\iint_S \left(x^3\bar{i} + y^3\bar{j} + z^3\bar{k}\right) \cdot \overline{ds}$$

where 'S' is the surface of the sphere :

$$x^2 + y^2 + z^2 = 16.$$

(*c*) Apply Stokes' theorem to calculate : [6]

$$\int_C 4y\,dx + 2z\,dy + 6y\,dz$$

where C is the curve of intersection of

$$x^2 + y^2 + z^2 = 6z \text{ and } z = x + 3.$$