Total No. of Questions—**12**] [Total No. of Printed Pages—**8+1**

Seat No.	

[4457]-120

S.E. (Electrical/Inst./Comp./I.T.) (II Sem.) EXAMINATION, 2013

(Common to Electrical/Inst./Comp./I.T.)

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Assume suitable data, if necessary.

(*vi*) Use of logarithmic tables, electronic pocket calculator is allowed.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $\left(D^4 - 1\right)y = \cos x \cosh x$

(*ii*) $\frac{d^2y}{dx^2} + 4y = x\sin x$

(*iii*) $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

(*iv*) $\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}$.

(*b*) A circuit consists of inductor of 2 henrys, a resistor of 4 ohms and capacitor of 0.05 farads. If $q = 0$, $i = 0$ at $t = 0$, find current and charge at time t. [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $(D^2 + 3D + 2)y = \sin(e^x)$

(*ii*) $(D^3 + D)y = \cos x$

(*iii*) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$

(*iv*) $\frac{d^2y}{dx^2} + 4y = \tan 2x$ (use method of variation of parameters).

(*b*) Solve : [5]

$$\frac{du}{dx} + v = \sin x$$

$$\frac{dv}{dx} + u = \cos x.$$

3. (*a*) If

$$u = \frac{-y}{x^2 + y^2}$$

find u such that $f(z) = u + iv$ is analytic function and determine $f(z)$ in terms of z. [5]

(*b*) Find the map of straight line $y = x$ under the transformation :

$$w = \frac{z-1}{z+1}.$$ [5]

(*c*) Evaluate :

$$\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4\cos\theta}\, d\theta.$$ [6]

Or

4. (*a*) Show that analytic function with constant amplitude is constant. [5]

(*b*) Find the bilinear transformation which maps the points $0, -1, i$ of z-plane to the points $2, \infty, \frac{1}{2}(5 + i)$ of w-plane respectively. [5]

(*c*) Evaluate :

$$\oint_C \frac{4z^2 + z}{(z-1)^2}\, dz$$

where C is contour $|z - 1| = 2$. [6]

5. (*a*) By considering Fourier sine transform of e^{-mx} $(m > 0)$, prove that :

$$\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + m^2} \, d\lambda = \frac{\pi}{2} e^{-mx}, \quad m > 0, x > 0.$$ [5]

(*b*) Solve the following integral equation :

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0.$$ [6]

(*c*) Find the z-transform of (any *two*) : [6]

(*i*) $f(k) = k5^k, \ (k \geq 0)$

(*ii*) $f(k) = 3^k \sinh \alpha k, \ k \geq 0$

(*iii*) $f(k) = \dfrac{2^k}{k}, \ k \geq 1.$

Or

6. (*a*) Find the Fourier cosine transform of : [5]

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1, \\ 2 - x, & 0 \leq x \leq 2, \\ 0, & x > 2. \end{cases}$$

(*b*) Find inverse z-transform of (any *two*) : [6]

(*i*) $\dfrac{3z^2 + 2z}{z^2 - 3z + 2}, \ 1 < |z| < 2$

(*ii*) $\dfrac{1}{(z-3)(z-2)}$

(*iii*) $\dfrac{z^2}{z^2 + 1}$ (use integral inversion method).

(c) Obtain $f(k)$, given that :

$$12f(k+2) - 7f(k+1) + f(k) = 0, \quad k \geq 0$$

$f(0) = 0,\ f(1) = 3.$ [6]

SECTION II

7. (a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the first four central moments, mean, standard deviation and β_1 and β_2. [9]

(b) From a group of 10 students marks obtained by each in papers of Mathematics and Applied Chemistry are as given as :

X : Marks in Mathematics	**Y : Marks in Applied Chemistry**
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

Calculate coefficient of correlation. [8]

Or

8. (*a*) A box contains 3 red balls, 2 white balls and 4 blue balls. Two balls are drawn successively from the box. Find the probability that they are drawn in the order red, white and blue if each ball is not replaced. [5]

(*b*) In a factory of razor blades, there is a chance of 1/500 for any blade to be defective. Blades are supplied in packets of 10. By using Poisson's distribution calculate approximate number of packets containing no defective and two defective blades in 10,000 packets. [6]

(*c*) On an average a box containing 10 articles is likely to have 2 defective. If we consider 100 boxes, how many of them are expected to have three or less defective ? [6]

9. (*a*) A curve is given by the equation $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$, find the angle between the tangents at $t = 1$ and $t = 2$. [5]

(*b*) Find the directional derivatives of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ towards the point $(3, -1, 2)$. [5]

(c) Solve any *two* : [6]

(i) If

$$\bar{F} = (x + y)\, i + (y + z)\, j + (z + x)\, \bar{k}$$

then show that curl curl $\bar{F} = 0$.

(ii) Show that :

$$\nabla \times \left(\frac{\bar{r}}{r^2}\right) = 0.$$

(iii) Show that :

$$\nabla \cdot \left(r^3 \bar{r}\right) = 6r^3.$$

Or

10. (a) Show that :

$$\bar{F} = \left(y e^{xy} \cos z\right) i + \left(x e^{xy} \cos z\right) j - e^{xy} \sin z\; k$$

is irrotational. Find scalar function ϕ such that $\bar{F} = \nabla\phi$. [5]

(b) Find the directional derivatives of $\phi = e^{2y} \cos xz$ at (0, 0, 0) in direction of tangent to the curve $x = a \cos t$, $y = a \sin t$, $z = at$ at $t = \frac{\pi}{4}$. [5]

(*c*) Solve any *two* : [6]

(*i*) Prove that :

$$\nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right] = \frac{3}{r^4}.$$

(*ii*) Show that :

$$\bar{E} \cdot \text{curl } \bar{E} = 0, \text{ if } \rho \bar{E} = \nabla \phi.$$

(*iii*) Show that :

$$\nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})}{r^{n+2}} \bar{r}.$$

11. (*a*) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r}, \bar{F} = 3x^2 \, \bar{i} + (2xz - y) \, \bar{j} + z \, \bar{k}$$

along the curve $x = t^2$, $y = 2t$, $z = 2t^2 - 1$ from $t = 0$ to $t = 1$. [5]

(*b*) Evaluate :

$$\iint_S (\nabla \times \bar{F}) \cdot \hat{n} \, dS$$

where S is the curve surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z = 2$, $\bar{F} = 3(x - y) \, \bar{i} + 2xz \, \bar{j} + xy \, \bar{k}$. [6]

(*c*) Evaluate :

$$\iint_S (x^3 \, \bar{i} + y^3 \, \bar{j} + z^3 \, \bar{k}) \cdot d\bar{S},$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$. [6]

Or

12. (*a*) Find the work done in moving a particle from (0, 0, 0) to (1, 1, 1) in a force field :

$$\overline{F} = yz\,\overline{i} + xz\,\overline{j} + xy\,\overline{k}.$$ [5]

(*b*) Evaluate :

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot d\overline{S}$$

for

$$\overline{F} = y\,\overline{i} + z\,\overline{j} + x\,\overline{k}$$

where S is surface of paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$. [6]

(*c*) Evaluate :

$$\iint_S \left(x\,\overline{i} + y\,\overline{j} + z^2\,\overline{k}\right) \cdot d\overline{S}$$

where S is the curve surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $z = 0$ and $z = 2$. [6]