

S.E. (COMP / IT / ELECTRICAL / INSTRUMENTATION)
ENGINEERING MATHEMATICS III

2008 Course

Time: 3 Hours

Max. Marks : 100

Instructions to the candidates:

- 1) In Section I attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.
- 2) In Section II attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.
- 3) Answers to the two sections should be written in separate answer books.
- 4) Neat diagrams must be drawn wherever necessary.
- 5) Figures to the right side indicate full marks.
- 6) Use of Calculator(non programmable) is allowed.
- 7) Assume Suitable data if necessary

SECTION I

- Q1) a) Solve the following (any three) [12]
- i) $(D^2 + 4)y = \sin x \cos 3x$
 - ii) $(2x + 1)^2 \frac{d^2y}{dx^2} - 2(2x + 1) \frac{dy}{dx} - 12y = 6x$
 - iii) $(D^2 + 9)y = \tan 3x$ (by variation of parameters)
 - iv) $(D^2 - 3D + 2)y = e^{-x}$
- b) A circuit consists of an inductance L & condenser of capacity C in series. An alternating emf $E \sin(\omega t)$ is applied to it. At time $t = 0$, the initial current and charge on the condenser being zero, find the current flowing in the circuit at any time t when $\omega \neq \omega_n$, if $\omega_n^2 = 1 / LC$ [5]

OR

- Q2) a) Solve the following (any three) [12]
- i) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = x \log x$
 - ii) $(D^2 - 4D + 4)y = \frac{e^{2x}}{x}$
 - iii) $(D^2 + 2D + 1)y = e^{-x} \log x$ (by variation of parameters)
 - iv) $\frac{x^2 dx}{y^3} = \frac{y^2 dy}{x^3} = \frac{dz}{z}$
- b) Solve : $\frac{du}{dx} + v = \sin x$, $\frac{dv}{dx} + u = \cos x$ [5]
 given at $x = 0$, $u = 1$ and $v = 0$
- Q3) a) Evaluate $\int_0^{1+i} (z^2 + 1) dz$ along the straight line. [5]
- b) If $u = \frac{1}{2} \log(x^2 + y^2)$, find v such that $f(z) = u + iv$ is analytic. Determine f(z) in terms of z. [6]

- c) Show that the transformation $\omega = (z - b)/(z + b)$ maps the right half of the z - plane into the unit circle $|\omega| < 1$. [5]

OR

- Q4) a) Evaluate using residue theorem, $\oint_c \frac{z+3}{(z-2)(z+1)^2} dz$ [6]

where c is the boundary of the square with vertices $(\pm 1.5, \pm 1.5)$

- b) If $f(z)$ is analytic, [5]

$$\text{show that } \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

- c) Find the bilinear transformation, which maps the point $-1, 0, 1$ of z - plane on to the points $0, i, 3i$ of the ω - plane [5]

- Q5) a) Using Fourier Integral representation [5]

$$\text{prove that } \int_0^\infty \frac{2\lambda \sin \lambda x}{\lambda^4 + 4} d\lambda = \frac{\pi}{2} e^{-x} \sin x$$

- b) Find the Fourier sine and cosine transforms of the following function: [6]

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$$

- c) Find the z transform of the following (**any two**) [6]

i. $f(k) = 3^k \cos(4k + 5), k \geq 0$

ii. $f(k) = \frac{5^k}{k}, k \geq 1$

iii. $f(k) = k2^k, k \geq 0$

OR

- Q6) a) Find inverse z - transform (**any two**) [6]

i. $F(z) = \frac{z}{(z-1)(z-2)}, |z| \geq 2$

ii. $F(z) = \frac{z^2}{z^2+1}$ by integral inversion method

iii. $F(z) = \frac{3z^2+2z}{z^2+3z+2}, 1 < |z| < 2$

- b) Solve the following: [6]

$$12f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0, f(0)=0, f(1) = 3$$

- c) Solve the Integral equation [5]

$$\int_0^\infty f(x) \cos \lambda x dx = \begin{cases} 1 - \lambda, & 0 \leq \lambda \leq 1 \\ 0, & \lambda \geq 1 \end{cases}$$

And hence show that

$$\int_0^{\infty} \frac{\sin^2 z}{z^2} dz = \frac{\pi}{2}$$

SECTION II

- Q7) a) Calculate the first four central moments of the given distribution. Is the distribution platykurtic? [9]

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| x | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
| f | 4 | 36 | 60 | 90 | 70 | 40 | 10 |

- b) The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y - 21 = 0$. The value of variance of x is 49. Find the correlation coefficient between x and y. Also find the standard deviation of y. [8]

OR

- Q8) a) The average number of misprints per page of a book is 2. Assuming the distribution of number of misprints to be Poisson, find the probability that a particular book is free from misprints and containing more than one misprint. [6]
- b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of distribution. [6]
- (Given Area corresponding $z = 0.5$ is 0.19 and $z = 1.41$ is 0.42)

- c) The mean and variance of Binomial distributions are 6 and 2 respectively. Find $p(r \geq 1)$. Where r is the number of successes in n trial. [5]

- Q9) a) If the directional derivative of $\phi = axy + byz + czx$ at (1,1,1) has maximum magnitude 8 in a direction parallel to z – axis, find the values of a, b and c. [6]

- b) Show that $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational and find scalar ϕ such that $\vec{F} = \nabla \phi$ [6]

- c) If \vec{u} and \vec{v} are irrotational vectors then, [4]
- prove that $\vec{u} \times \vec{v}$ is solenoidal vector.

OR

- 10) a) Find the directional derivatives of $\phi = \sqrt{3} e^{x+y+z}$ at (0, 0, 0) along a line equally inclined with co – ordinate axes. [5]

- b) Determine f(r) such that $f(r)\vec{r}$ is solenoidal. [3]

- c) Show that **(any two)** [8]

$$\text{i. } \nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^2} \right) = \frac{\bar{a}}{r^2} - \frac{2(\bar{a} \cdot \bar{r})}{r^4} \bar{r}$$

$$\text{ii. } \nabla \times [\bar{a} \times (\bar{b} \times \bar{r})] = \bar{a} \times \bar{b}$$

iii. For a scalar function u and v , show that

$$\nabla \cdot (u \nabla \nabla - \nabla \nabla u) = u \nabla^2 v - \nabla \nabla^2 u$$

11) a) Using Green's theorem, Evaluate $\oint_C (e^y \bar{i} + x(1 + e^y) \bar{j}) \cdot d\bar{r}$ for a closed curve C given by $\frac{x^2}{36} + \frac{y^2}{49} = 1, z = 0$ [6]

b) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot \hat{n} ds$ where 's' is the curved surface of the paraboloid $x^2 + y^2 = 4z$ bounded by the plane $z = 4$, where $\bar{F} = 3(x - y) \bar{i} + 2xz \bar{j} + xy \bar{k}$. [6]

c) Show that $\iiint_V \frac{2 dv}{r} = \iint_S \frac{\bar{r} \cdot \hat{n}}{r} ds$ [5]

OR

12) a) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ for $\bar{F} = 3x \bar{i} + (x - y) \bar{j} + z \bar{k}$, along the curve $x = 3t, y = t, z = t^2$ from $t = 0$ to $t = 1$. [6]

b) Apply Stokes theorem to evaluate $\int_C 4y dx + 2z dy + 6y dz$, where c is the curve of intersection of $x^2 + y^2 + z^2 = 9$ and $x + z = 0$ [6]

c) If $\nabla \cdot \bar{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$ and $\nabla^2 \bar{A} = \frac{1}{c^2} \frac{\partial^2 \bar{A}}{\partial t^2}$ then [5]

$$\text{Show that } \bar{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \bar{A}}{\partial t}, \bar{H} = \nabla \times \bar{A}$$

$$\text{are solution of Maxwell's equation } \nabla \times \bar{H} = \frac{1}{c} \frac{\partial \bar{E}}{\partial t}$$