

Total No. of Questions—12]

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[4657]-79

S.E. (Computer/IT) (First Sem.) EXAMINATION, 2014

DISCRETE STRUCTURES

(2008 PATTERN)

Time : Three Hours

Maximum Marks : 100

N.B. :— (i) Section I : Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.

(ii) **Section II :** Attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(iii) Answers to the two Sections should be written in separate answer-books.

(iv) Neat diagrams must be drawn wherever necessary.

(v) Assume suitable data, if necessary.

SECTION I

1. (a) Show that :

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$ by mathematical induction.

[4]

P.T.O.

(b) Among the integers 1 to 1000 :

(i) How many of them are not divisible by 3, nor by 5, nor by 7 ?

(ii) How many are not divisible by 5 and 7 but divisible by 3 ? [6]

(c) Obtain the CNF and DNF of the following formulae given below :

(i) $p \cap (p \rightarrow q)$

(ii) $\sim (p \vee q) \iff (p \cap q)$. [6]

Or

2. (a) Construct truth table to determine whether each of the following is tautology or a contradiction :

(i) $(p \cap q) \cap \sim (p \vee q)$

(ii) $(p \rightarrow q) \leftrightarrow (q \vee \sim p)$. [4]

(b) Negate the following in such a way that the symbol \sim does not appear outside the bracket :

(i) $\forall x (x^2 > x)$

(ii) $\exists x (x^2 = 2)$. [4]

(c) For the universe of all integers let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x)$ be the following statements :

$P(x)$: $x > 0$

$Q(x)$: x is even

$R(x)$: x is a perfect square

$S(x)$: x is divisible by 4

$T(x)$: x is divisible by 5

Write the following statements in symbolic form :

(i) At least one integer is even.

(ii) There exists a positive integer that is even.

(iii) If x is even, then x is not divisible by 5.

(iv) No even integer is divisible by 5.

(v) There exists an even integer divisible by 5.

(vi) If x is even and x is perfect square, then x is divisible
by 4. [6]

(d) Define subset and powerset along with example. [2]

3. (a) Let G be the set of all non-zero real numbers and let :

$$a * b = \frac{ab}{2}.$$

Show that $(G, *)$ is an abelian group. [6]

- (b) If R is a ring such that $a^2 = a, \forall a \in R$, prove that :

$$a + a = 0, \forall a \in R. [6]$$

- (c) Define each of the following :

(i) Groups

(ii) Rings

(iii) Normal subgroups

(iv) Homomorphism of groups. [4]

Or

4. (a) Define each of the following :

(i) Cyclic group

(ii) Subgroups

(iii) Cosets

(iv) Subrings. [4]

(b) Let G be a group for a fixed element G , let $Gx = \{a \in G : ax = xa\}$. Show that Gx is a subgroup of G for all $x \in G$. [6]

(c) Let $G = \{e, a, a^2, a^3, a^4, a^5\}$ be a group under the operation of $a^i a^j = a^r$ where $i + j = r \pmod{6}$. Prove that G and Z_6 are isomorphic. [6]

5. (a) Let

$$R = \{(1, 4) (2, 1) (2, 5) (2, 4) (4, 3) (5, 3) (3, 2)\}.$$

Use Warshall's algorithm to find the matrix of transitive closure. [6]

(b) Show that if 7 colours are used to paint 50 bicycles, at least 8 bicycles will be the same colour. [6]

(c) Solve the recurrence relation :

$$a_n = 5a_{n-1} - 6a_{n-2}, \quad n \geq 2$$

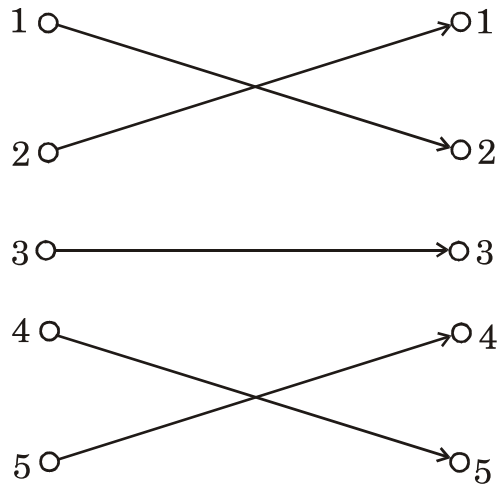
given $a_0 = 1, a_1 = 4$. [6]

Or

6. (a) Let

$$A = \{1, 2, 3, 4, 5\}, g : A \rightarrow A$$

is as shown in figure.



Find the composition gog , $go(gog)$. Determine whether each is one-to-one or onto function. [6]

(b) Let A is set of factors of positive integer m and relation is divisibility on A .

$$\text{i.e. } R = \{(x, y) \mid x, y \in A, x \text{ divides } y\}$$

For $m = 45$, show that $\text{POSET}(A, \leq)$ is a lattice. Draw Hasse diagram and give join and meet for the lattice. [6]

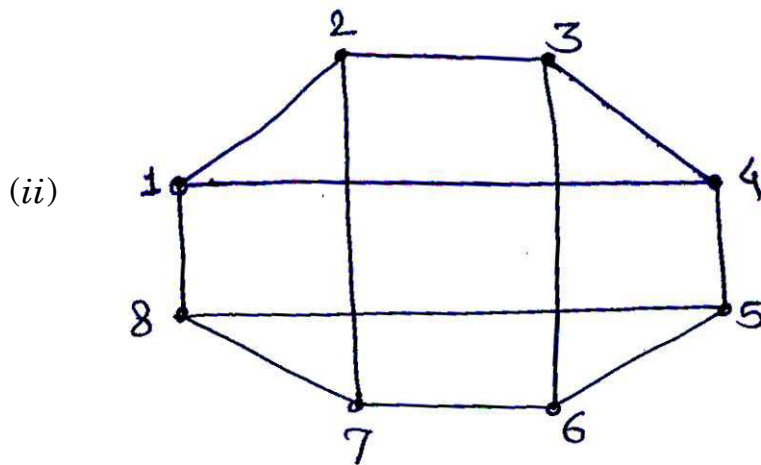
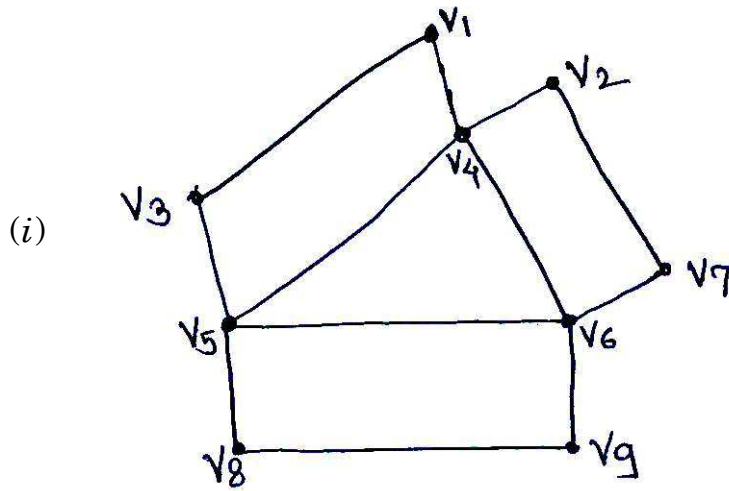
(c) Let R be a binary relation on the set of all positive integers such that :

$$R = \{(a, b) \mid a - b \text{ is an odd positive integer}\}$$

Is relation R reflexive, symmetric, antisymmetric, transitive, equivalence and partial ordering relation ? [6]

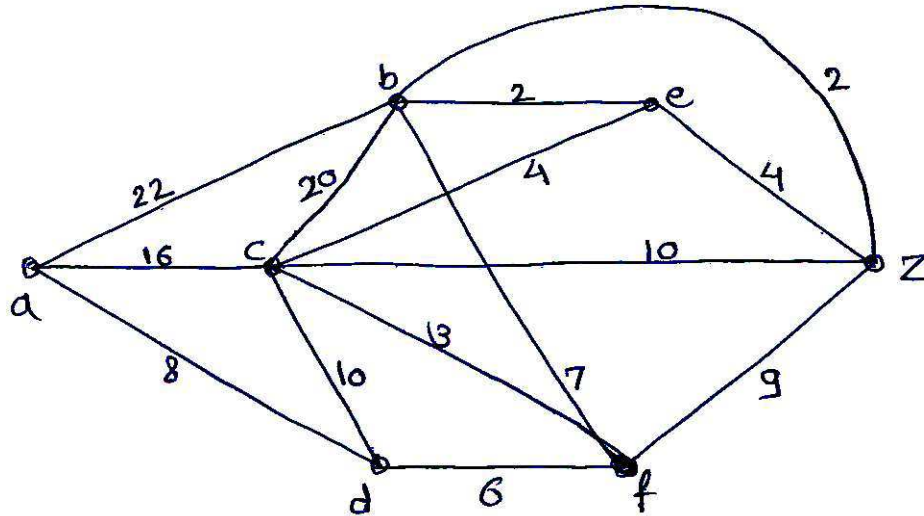
SECTION II

7. (a) Determine whether or not each of the graph is bipartite. In each case give the bipartition set or explain why the graph is not bipartite. [6]



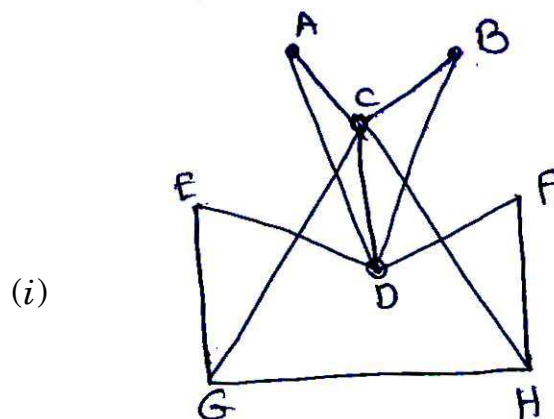
- (b) Show that in a connected planar linear graph with 6 vertices and 12 edges, each of the region is bounded by 3 edges. [4]

- (c) Use Dijkstra's algorithm to find the shortest path from a to z . [8]

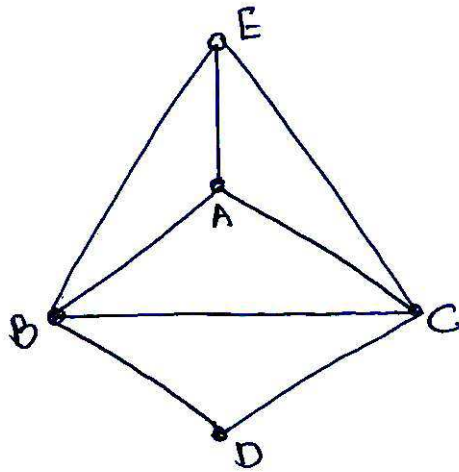


Or

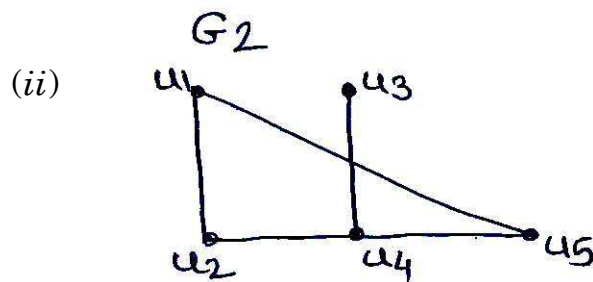
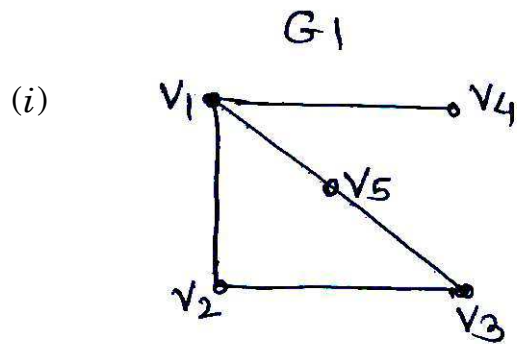
8. (a) Determine whether the following graphs are Hamiltonian or Eulerian. [4]



(ii)



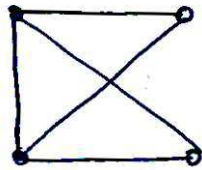
(b) Find the adjacency matrices A_1 and A_2 of the graph G_1 and G_2 . [4]



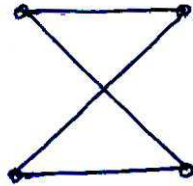
(c) For each of the ten pairs of graphs :

[*i-ii*, *i-iii*, *i-iv*, *i-v*, *ii-iii*, *ii-iv*, *ii-v*, *iii-iv*, *iii-v*, *iv-v*]
label the graphs so as to exhibit an isomorphism or explain why the graphs are not isomorphic. [10]

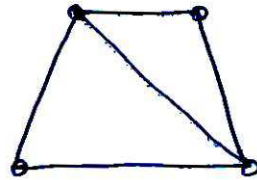
(i)



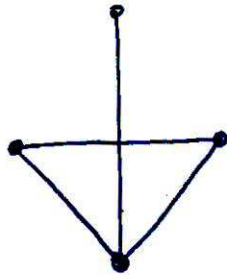
(ii)



(iii)



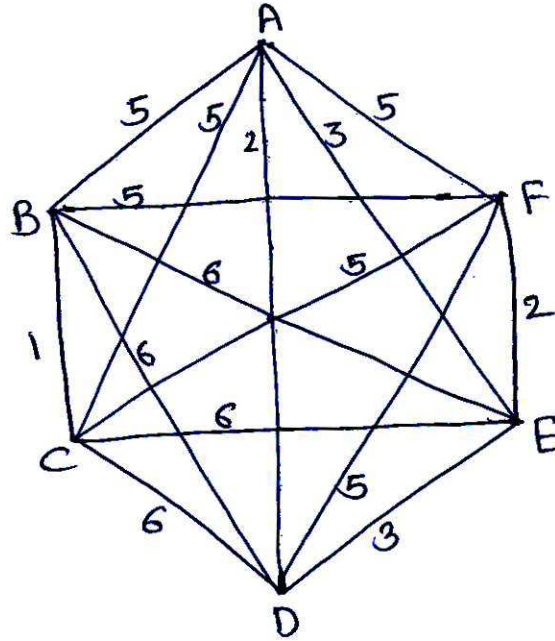
(iv)



(v)



9. (a) Find minimal spanning tree using Kruskal's algorithm. [6]



- (b) Construct an optimal tree for the weights :

8, 9, 10, 11, 13, 15, 22.

Find the weight of the optimal tree. [6]

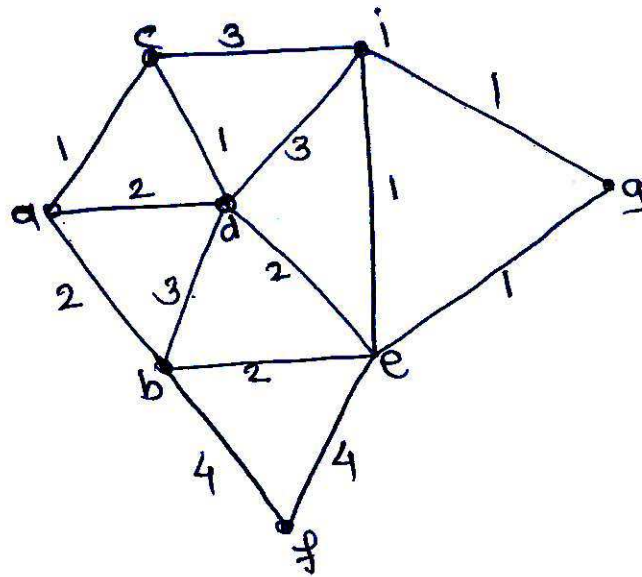
- (c) Define the following terms with example : [4]

(i) Binary Tree

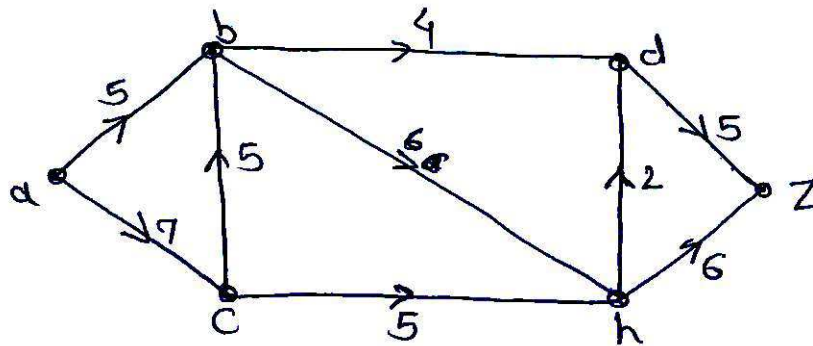
(ii) Optimal Binary Tree.

Or

10. (a) Use Prim's algorithm to construct minimal spanning tree starting from vertex a . [6]



- (b) Define spanning tree and minimum spanning tree along with example. [2]
- (c) Find maximum flow in the transport network using labelling procedure. Determine the corresponding minimum cut. [8]



11. (a) A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{3}$ and that of wife's selection is $\frac{1}{2}$. What is the probability that :
- (i) both of them will be selected ?
 - (ii) only one of them will be selected ?
 - (iii) none of them will be selected ? [6]
- (b) A group of people is comprised of six from Maharashtra, seven from Gujrat and eight from Goa.
- (i) In how many ways can a committee of six be formed with two people from each state ?
 - (ii) In how many ways can a committee of seven be formed with at least two people from each state ? [4]
- (c) There are three bags : Bag A contains 1 white, 2 red, 3 green balls; Bag B contains 2 white, 3 red, 1 green ball; Bag C contains 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. They are found to be 1 white and 1 red. Find the probability that two balls come from bag B. [6]

Or

12. (a) A company purchased 10,000 transistors; 5000 from supplier A, 3000 from supplier B and 2000 from supplier C. It is known that 2% of supplier A are defective, 4% of supplier B are defective and 5% of supplier C are defective :

(i) If transistor from 10,000 is selected at random, what is the probability that it is defective ?

(ii) If transistor selected at random is defective, what is the probability that it is from supplier A ?

(iii) Given that transistor selected at random is not from supplier A, what is the probability that it is defective ? [6]

(b) A coin is tossed ten times and the sequence of heads and tails observed.

(i) How many different sequences are possible ?

(ii) In how many of these sequences are there exactly four heads ? [4]

- (c) In a bolt factory there are four machines A, B, C, D manufacture 20%, 25%, 10% and 45%. Of the total bolts respectively 2% of the bolts manufactured by A, 4% by B, 2% by C and 5% by D are found to be defective. A bolt is chosen at random and is found to be defective. What is the probability that it is manufactured by C ? [6]