

Total No. of Questions—12]

[Total No. of Printed Pages—8+1

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[4757]-190

S.E. (I Sem.) EXAMINATION, 2015
(Common to Computer Engineering and IT)
DISCRETE STRUCTURES
(2008 PATTERN)

Time : Three Hours

Maximum Marks : 100

- N.B. :-**
- (i) Answers to the two Sections should be written in separate answer-books.
 - (ii) In Section I attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.
 - (iii) In Section II attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.
 - (iv) Neat diagrams must be drawn wherever necessary.
 - (v) Figures to the right indicate full marks.
 - (vi) Assume suitable data, if necessary.

SECTION I

1. (a) Determine whether each of the following statements is true or false. Briefly explain your answer : [8]
- (i) $A \cup P(A) = P(A)$
 - (ii) $\{A\} \cup P(A) = P(A)$
 - (iii) $\{A\} \cap P(A) = A$
 - (iv) $A - P(A) = A$.

P.T.O.

- (b) Obtain the conjunctive normal form of : [8]
- (i) $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$
- (ii) $(p \wedge q) \vee (\sim p \wedge q \wedge r)$.

Or

2. (a) Give truth table and comment whether tautology or contradiction : [8]
- (i) $\neg(\neg P \vee \neg Q)$
- (ii) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \vee R)$
- (iii) $(P \rightarrow Q) \wedge (Q \rightarrow P)$.
- (b) To describe various restaurants in the city, we let p denotes the statement “The food is good”, q denotes the statement “The service is good”, r denotes the statement “The rating is three-star”. Write the following statement is symbolic form : [6]
- (i) Either the food is good or the service is good or both.
- (ii) Either the food is good or the service is good but not both.
- (iii) The food is good while the service is poor.
- (iv) It is not the case that both the food is good and the rating is three-star.
- (v) If both the food and services are good, then the rating will be three-star.
- (vi) It is not true that a three-star rating always means good food and good service.
- (c) Define the term countably infinite set and uncountably infinite set. [2]

3. (a) Define with examples : [8]
- (i) Subgroup
 - (ii) Cyclic Group
 - (iii) Field
 - (iv) Integral domain.
- (b) Prove the following results for the group G : [6]
- (i) The identity element is unique.
 - (ii) Each a in G has a unique inverse a^{-1} .
 - (iii) $ab = ac$ implies $b = c$.
- (c) What is homomorphism and automorphism in an algebraic system ? [2]

Or

4. (a) A central groupoid is an algebraic system $(A, *)$, where $*$ is a binary operation such that $(a * b) * (b * c) = b$. For all a, b, c belongs to A . Such that : [8]
- (i) $a * ((a * b) * c) = a * b$
 - (ii) $(a * (b * c)) * c = b * c$.
- (b) Let $Z = \{0, 1, 2, \dots, n - 1\}$. Let \diamond be a binary operation such that $a \diamond b =$ remainder of $a.b$ divided by n . Construct a table of $n = 4$. Is (Z_4, \diamond) a groupoid, monoid, semi-group and abelian group. [8]

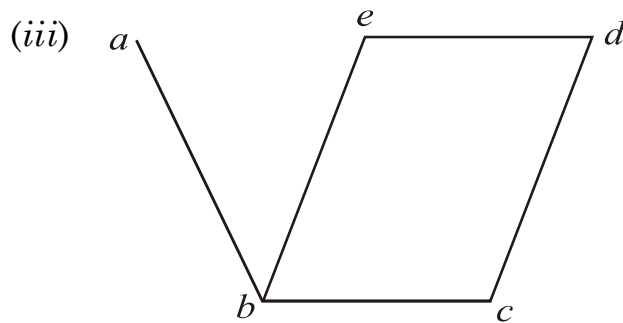
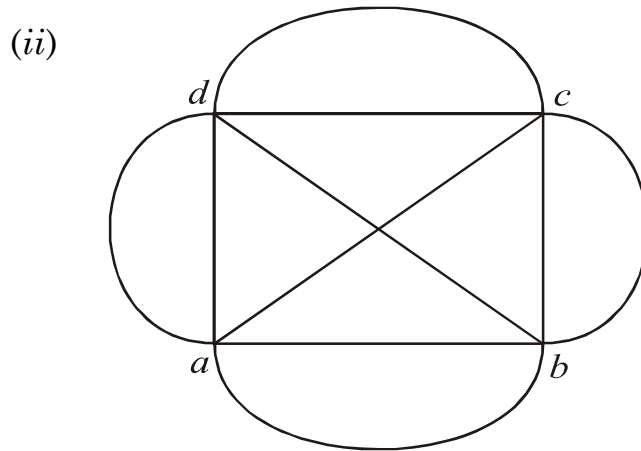
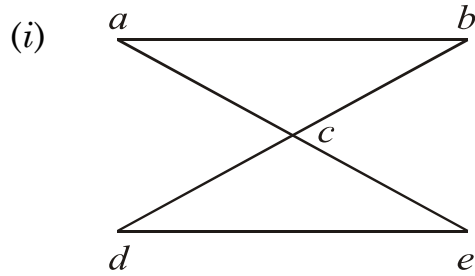
5. (a) Let R be a binary relation on the set of all strings of 0's and 1's such that $R = \{(a, b) \mid a \text{ and } b \text{ are strings that have the same number of 0's}\}$: [8]
- (i) Is R reflexive ?
 - (ii) Is R symmetric ?
 - (iii) Is R antisymmetric ?
 - (iv) Is R transitive ?
 - (v) Is R equivalence relation ?
 - (vi) Is R a partial ordering relation ?
- (b) Find the transitive closure of R by Warshall's algorithm :
 $A = \{\text{set of integers } \leq 10\}$, $B = \{(a, b) \mid a \text{ divides } b\}$. [8]
- (c) Let $X = \{1, 2, 3\}$ and f and g , be function from X to X given by : [2]
 $f = \{(1, 2), (2, 3), (3, 1)\}$, $g = \{(1, 2), (2, 1), (3, 3)\}$.
 Find :
 (i) $f \circ g$
 (ii) $g \circ f$.

Or

6. (a) For a given set A consider $R = \{(x, y) \mid x \in P(A), y \in P(A), x \dot{\subseteq} y\}$. Show that R is a partial ordering relation. What is the length of the longest chain in the poset $\{P(A), R\}$? [8]
- (b) Let $A = \{1, 2, 3, 4, 6, 12, 16, 36\}$ and let R be the relation on A defined by x divides y . Draw the Hasse diagram. Compare it with digraph. Determine whether R is a reflexive, transitive and symmetric. [8]
- (c) Let $A = \{2, 3, 4, 6\}$ and let aRb if a divides b . State R is a partial order. [2]

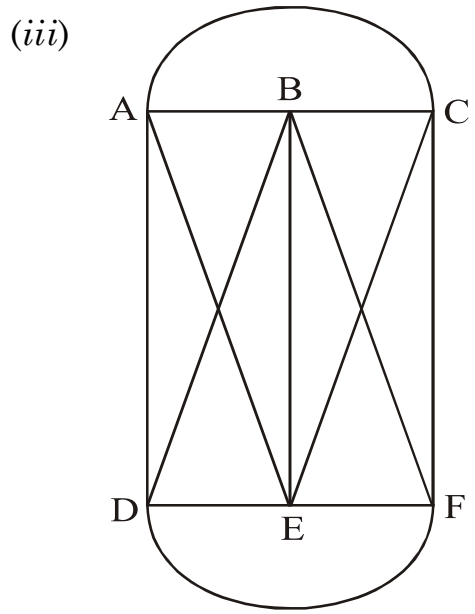
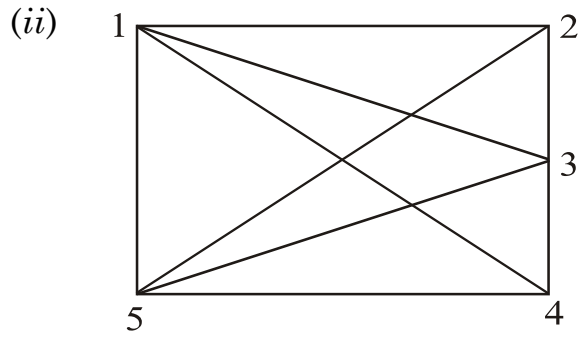
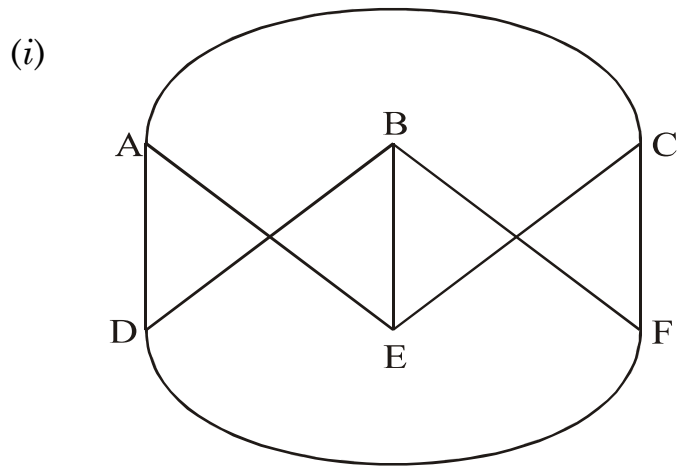
SECTION II

7. (a) Which of the following graphs have a Euler circuit or path or Hamiltonian cycle ? Write the path or circuit : [6]



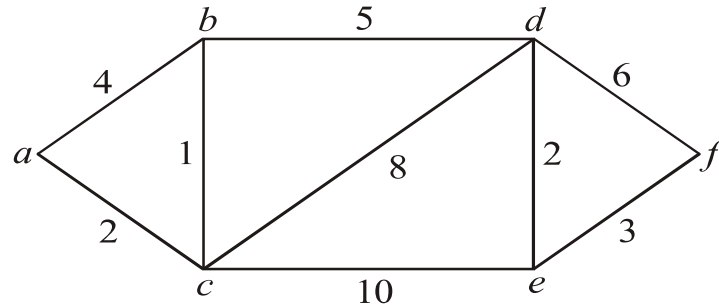
- (b) Define graph and multigraph. Give any *two* applications of graph and represent them in graph notation. [6]

(c) Identify whether the graphs given are planar or not. Draw planar representation if possible : [6]

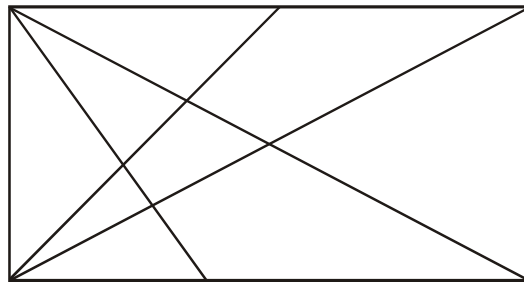


Or

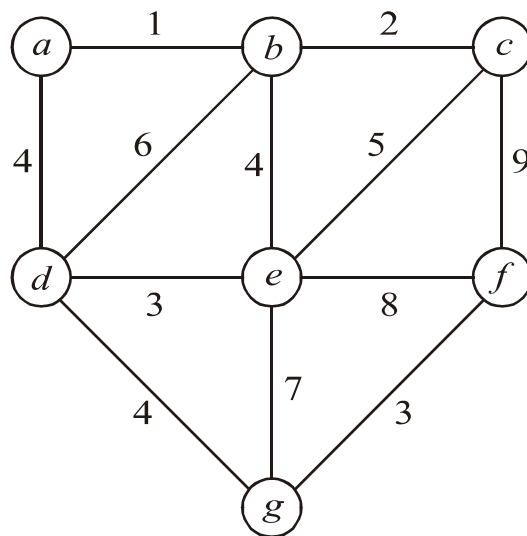
8. (a) State and prove Euler's formula for a connected planar graph of order n , size e and with f faces. [6]
 (b) Use Dijkstra algorithm to find the shortest path from a to f . [8]



- (c) Draw isomorphic graph of a graph shown in the following figure but no crossover of edges. [4]



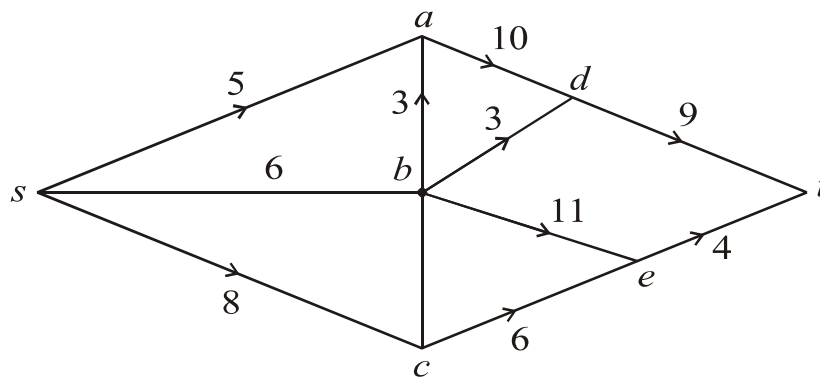
9. (a) Using Prim's algorithm construct minimal spanning tree starting at vertex a . [6]



- (b) Draw a binary search tree for input data 200, 100, 300, 50, 150, 250, 400, 10, 75, 125, 175. Which is a root, leaf nodes and interior nodes. [6]
- (c) Construct the expression tree for the following expression :
 $(3 - (2(-11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$.
 Also evaluate expression. [4]

Or

10. (a) Use labeling procedure to find maximum flow in transport network shown below. Define corresponding minimal cuts. [8]



- (b) Use Huffman coding to encode the following symbol with the frequencies listed A : 0.08, B : 0.010, C : 0.12, D : 0.15, E : 0.20, F : 0.35. What is average number of bits used to encode the character ? [8]
11. (a) If repetitions are not permitted, how many four digit numbers can be formed from digits 1, 2, 3, 7, 8 and 5. [6]

(b) Show that : [6]

$$c(2n, 2) = 2c(n, 2) + n^2.$$

(c) How many seven letter words can be formed using the letters of the word BENZENE ? [4]

Or

12. (a) Three students A, B and C are swimming in the race. A and B have same probability of winning and each is twice as likely to win as C. Find probability that : [6]

(i) B wins

(ii) C wins

(iii) B or C wins.

(b) A woman has 11 friends of them six are women : [6]

(i) In how many ways can she invite three or more ?

(ii) In how many ways can she invite three or more of them if she wants same number of men and women (including herself) ?

(c) A coin is tossed 3 times. Find the probability that there will appear : [4]

(i) Three heads

(ii) Exactly 2 heads

(iii) No heads.