Seat No:

210241 - Discrete Structures

S. E. (Common for Computer Engg./Information Technology)

2008 Pattern (Semester-I)

Duration: - 3 Hours.   **Max. Marks: - 100**

Instructions to the candidates

1. *Answer Question 1 or 2, 3 or 4, and 5 or 6 from section - I and Question 7 or 8, 9 or 10, and 11 or 12 from section – II.*
2. *Answers to the two sections should be written in separate answer books.*
3. *Neat diagrams must be drawn wherever necessary.*
4. *Figures to the right indicate full marks.*
5. *Assume suitable data if necessary.*

SECTION – I

Q1.   A   Use mathematical induction to show that: [6]
$$1/1.2 + 1/2.3 + 1/3.4+ \ldots + 1/n\,(n{+}1) = n/(n{+}1) \;\forall\; n \geq 1$$

B   Over the universe of book defined propositions   [4]

      B(x): x has blue cover

      M(x): x is maths book

      I(x): x published in India

      Translate the following

      i) $\forall x (M(x) \wedge I(x) \rightarrow B(x))$
      ii)   There are maths books published outside India.

C   100 of the 120 engineering students in a college take part in at least one of the activity   [6]
group discussion, debate and quiz. Also 65 participate in group discussion, 45
participate in debate, 42 participate in quiz, 20 participate in group discussion and
debate, 25 participate in group discussion and quiz 15 participate in debate and quiz.

Find the number of students who participate in:

    i)   All three activities

    ii)   Exactly one of the activity

<div align="center">OR</div>

Q2.   A   With the help of mathematical induction prove that,   [6]

$$1^2 + 3^2 + 5^2 + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

    B   What is Multiset? With reference to multiset define the following with example   [6]
      i)   Multiplicity
      ii)   Union
      iii)   Intersection
      iii)   Difference

    C   Explain the concept of countable finite and infinite set with example   [4]

Q3.   A   let G={even, odd}and binary operation $\oplus$ be defined as   [6]

$\oplus$	Even	Odd
even	even	Odd
odd	odd	even

Show that (G, $\oplus$) is a group.

    B   Define the following term with reference to Group theory with suitable example.   [6]
      i)   Homomorphism
      ii)   Automorphism
      iii)   Isomorphism

    C   Let (A,*) be a group ,Show that (A,*) is an abelian group if and only if a^2 *   [4]
      $b^2 = (a*b)^2$

<div align="center">OR</div>

Q4.   A   Prove that the set Z of all integers with binary operation * defined by a*b =a+b+1 $\forall$ a,   [6]
      b $\in$ Z is an abelian group.

    B   Define the following terms with reference to Group theory.   [4]
      i)   Abelian group
      ii)   Cyclic group

    C   Consider the Group G= {1,2,3,4,5,6} under multiplication modulo 7   [6]
      1. Find the multiplication table of G.
   2. Find 2^{-1} , 3^{-1} , 6^{-1}
      3. Is G Cyclic ?

Q5.　A　Let A,B,C are set of real numbers . f : A →>B; g : B→ C be defined by f(a) = a-1; g(b)　[8]
= b ^{2} .Find
1. (fog)(2)
2. (gof)(2)
3. (gof)(x)
4. (fog)(x)
5. (fof)(y)
6. (gog)(y)

　　B　Let x = {2,3, 6, 12, 24, 36} x ≤ y if x divides y. Find　[8]
1. Maximal element
2. Minimal element
3. Chain
4. Antichain

　　C　What is POSET ?　[2]

<center>OR</center>

Q6.　A　Let A={1,2,3,4,5} and   R={(1,4),(2,1), (2,5),(2,4),(4,3),(5,3),(3,2) }. Find the　[8]
transitive closure by using Warshall's algorithm .

　　B　Let A={1,2,3,4,5,6} and Π =[{1,2},{3,4,5},{6}] .Find the equivalence relation　[8]
determined by Π .Also draw its diagraph.

　　C　What is the difference between relation & function ?　[2]

<center>**SECTION – II**</center>

Q7.　A　State and prove Euler's formula for a connected planar graph　[6]
of order n, size e and with f faces

　　B　Find the shortest path from A to Z by using Dijkstra's Shortest path algorithm.　[8]

<center>3</center>
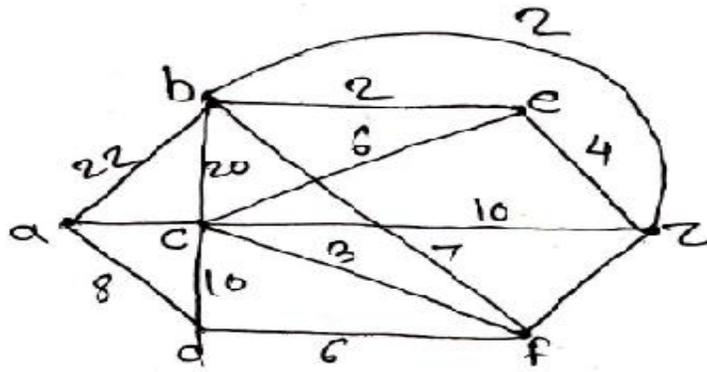
    C   List the application of graph theory.   [2]

OR

Q8.   A Define the following with an example with respect to graph theory.   [8]
        i) Isomorphic graph
        ii) Bipartite graph
        iii) Self complementary graph
        iv) Planar graph

      B Define Eulerian path and Eulerian circuit ? find under what conditions Km,n, the   [8]
         complete bipartite graph will have an Eulerian circuit ? Explain all cases.

Q9.   A Define the following terms with suitable example :   [6]
        (*i*) Rooted tree
        (*ii*) Spanning tree
        (*iii*) Level of the tree

      B Use Huffman coding to encode the following symbol with the frequencies listed.   [6]
         A:0.08,B:0.010,C:0.12,D:0.15,E:0.20,F:0.35.List the prefix code.

      C Draw a binary tree for input data 200,100,300,50,150,250,400,10,75,125,175.Identify [4]
         the root, leaf & interior nodes.

OR

Q10. A Find the minimum spanning tree of the given figure by using Prim's algorithm   [6]
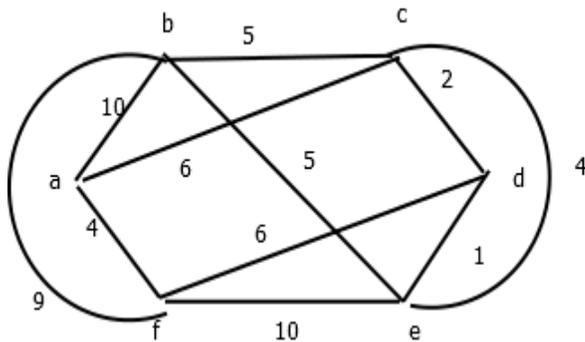
      B A binary tree has 10 nodes. The inorder and preorder   [6]
         traversals of the trees are as shown below. Construct the binary tree.
         Inorder : A B C E D F J G I H
         Preorder : J C B A D E F I G H

4

C Construct the expression tree for the following expression. [4]
$(3 - (2(- 11 - (9 - 4)))) / (2 + (3 + (4 + 7)))$.
Also evaluate the expression.

Q11. A If repetition is not allowed how many 4 digits number can be formed with the digits [6]
1,2,3,4,5,7,8?
i) How many are less than 5000?
ii) How many even digits can be formed?
iii) How many odd digits can be formed?

B A fair coin is tossed 6 times. Find the probability [6]

  i) of getting exactly two heads.
  ii) of getting exactly no heads.
  iii) of getting one or more heads.

C How many Auto license plates can be made if it is of the form of two letters followed [6]
by 4 digits? Also
  i) Find for the case if repetitions are not allowed.
  ii) Find for the case if only digits are repeated.
  iii) Find for the case if only letters are repeated.

<div align="center">OR</div>

Q12. A In how many ways 9 people can be seated at a round table if [6]
i) They can sit anywhere?
ii) 2 particular person must not sit next to each other?

B Find the number of distinct permutation that can be formed from the given words [4]
RADAR   ii) UNUSUAL

C The probability that A hits the target is 1/3 and the probability that B hits the target is [8]
1/5.They both fire at the target .Find the probability that

  i) A does not hit the target.
  ii) Both hit the target .
  iii) One of them hit the target.
  iv) Neither hit the target.