Total No. of Questions—8] [Total No. of Printed Pages—4

Seat No.	

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F.E. EXAMINATION, 2017

ENGINEERING MATHEMATICS–II

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Attempt *four* questions : Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6**, Q. No. **7** or Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programmable calculator is allowed.

(*v*) Assume suitable data if necessary.

1. (A) Solve the following differential equations : [8]

(*i*) $x^4 \frac{dy}{dx} + x^3 y = \sec(xy)$

(*ii*) $\frac{dy}{dx} = \frac{1 + y^2 + 3x^2 y}{1 - 2xy - x^3}$

(B) A body starts moving from rest is opposed by a force per unit mass of value cx and resistance per unit mass of a value bv^2, where x and v are the displacement and velocity of the particle at that instant. Show that the velocity of the particle is given by : [4]

$$v^2 = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}$$

P.T.O.

Or

2. (A) Solve : [4]

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.$$

(B) Solve the following : [8]

(*i*) Water at temperature 100°C cools in 10-minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes.

(*ii*) A resistance of 100 Ω, an inductance of 0.5 henry are connected in a series with battery of 20 volts. Find the current in a circuit as a function of time t.

3. (*a*) Find Fourier series to represent the function $f(x) = x$ in $-\pi < x < \pi$ and $f(x) = f(x + 2\pi)$. [5]

(*b*) Evaluate : [3]

$$\int_0^\infty \sqrt{y}.\ e^{-\sqrt{y}}\, dy$$

(*c*) Trace the curve (any *one*) : [4]

(*i*) $y^2 (x^2 - 1) = x$

(*ii*) $r = a(1 + \cos\theta)$.

Or

4. (*a*) If :

$$I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \ d\theta,$$

prove that :

$$I_n = \frac{1}{n-1} - I_{n-2}.$$ [4]

(*b*) Prove that :

$$\int_0^1 \frac{x^a - x^b}{\log x}\,dx = \log\left(\frac{a+1}{b+1}\right),\quad a > 0,\ b > 0 \qquad [4]$$

(*c*) Find the length of the arc of cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between two consecutive cusps. [4]

5. (*a*) Find the centre and radius of the circle which is an intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ by the plane $x + 2y + 2z = 15$. [5]

(*b*) Find the equation of the right circular cone which passes through the point (1, 1, 2) & has its axis along the line $6x = -3y = 4z$ and vertex at (0, 0, 0). [4]

(*c*) Find the equation of a right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, – 3, 6. [4]

Or

6. (*a*) Show that the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere $x^2 + y^2 + z^2 - y - 2z - 14 = 0$. [5]

(*b*) Find the equation of a right circular cone whose vertex is at (1, 2, 3) and axis has direction ratios (2, –1, 4) and semivertical angle 60º. [4]

(*c*) Find the equation of the right circular cylinder of radius 3 whose axis is the line

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1} \qquad [4]$$

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7. Attempt any *two* of the following :

(*a*) Evaluate

$$\iint \frac{x^2 y^2 \, dxdy}{x^2 + y^2}$$

where R is annulus between $x^2 + y^2 = 4$, $x^2 + y^2 = 9$. [6]

(*b*) Evaluate

$$\iiint \left(x^2y^2 + y^2z^2 + z^2x^2\right) dxdydz$$

throughout the volume of sphere $x^2 + y^2 + z^2 = a^2$. [7]

(*c*) Find the moment of inertia of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. [6]

Or

8. Attempt any *two* of the following :

(*a*) Find the total area included between the two cardioids $r = a\ (1 + \cos\ \theta)$ and $r = a(1 - \cos\theta)$. [6]

(*b*) Find the volume cut-off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1$ by the plane $z = 0$. [7]

(*c*) Find the C.G. of an area of the cardioid : $r = a\ (1 + \cos\ \theta)$. [6]