

Total No. of Questions—8]

[Total No. of Printed Pages—6

Seat No.	
-------------	--

[5151]-101

F.E. EXAMINATION, 2017
ENGINEERING MATHEMATICS-I
(2015 PATTERN)

Time : Three Hours

Maximum Marks : 50

N.B. :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3
or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or
Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Show that system of Linear equations is consistent. Find
solution : [4]

$$x + 2y + 3z = 6$$

$$2x + 3y = 11$$

$$4x + y - 5z = -3.$$

(b) Find eigen values and eigen vectors of the matrix : [4]

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

(c) Prove that : [4]

$$(\cosh x - \sinh x)^n = \cosh nx - \sinh nx.$$

P.T.O.

Or

2. (a) Are the following vectors are linearly dependent ? If so find relation : [4]

$$X_1 = (3, 2, 7),$$

$$X_2 = (2, 4, 1),$$

$$X_3 = (1, -2, 6).$$

- (b) If α, β are roots of equation $x^2 - 2x + 4 = 0$, prove that : [4]

$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

- (c) If [4]

$$(a+ib)^p = m^{x+iy},$$

prove that :

$$\frac{y}{x} = \frac{2 \tan^{-1} \frac{b}{a}}{\log(a^2 + b^2)}.$$

3. (a) Test the convergence of the series (any one) : [4]

(i) $\sum \frac{x^n}{a + \sqrt{n}}$

(ii) $\frac{1}{\sqrt{5}} - \frac{1}{2\sqrt{6}} + \frac{1}{3\sqrt{7}} - \dots$

- (b) Show that : [4]

$$\log \left[\frac{1+e^{2x}}{e^x} \right] = \log 2 + \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} - \dots$$

- (c) Find the n th derivative of : [4]

$$y = e^x \cos x \cdot \cos 2x.$$

Or

4. (a) Solve any one :

[4]

$$(i) \lim_{x \rightarrow 0} \left[\frac{\pi}{4x} - \frac{\pi}{2x(e^{\pi x} + 1)} \right]$$

$$(ii) \lim_{x \rightarrow 0} \left(\frac{2^x + 5^x + 7^x}{3} \right)^{1/x}$$

(b) Using Taylor's theorem expand $x^3 - 2x^2 + 3x + 1$ in powers of $(x - 1)$. [4]

(c) If $y = e^{a \sin^{-1} x}$, prove that : [4]

$$(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} - (n^2 + a^2)y_n = 0.$$

5. Solve any two :

(a) Find the value of n for which $z = t^n e^{-t^2/4t}$ satisfies the partial differential equation : [6]

$$\frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial z}{\partial r} \right) \right] = \frac{\partial z}{\partial t}$$

(b) If

$$T = \sin \left(\frac{xy}{x^2 + y^2} \right) + \sqrt{x^2 + y^2} + \frac{x^2 y}{x + y},$$

find the value of $x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y}$. [7]

(c) If $z = f(x, y)$, where

$$x = u \cos \alpha - v \sin \alpha,$$

$$y = u \sin \alpha - v \cos \alpha,$$

where α is constant, show that : [6]

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2.$$

Or

6. Solve any two :

(a) If [6]

$$x^2 = a\sqrt{u} + b\sqrt{v} \quad \text{and}$$

$$y^2 = a\sqrt{u} - b\sqrt{v}$$

where a and b are constants, prove that :

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u.$$

(b) If

$$u = \tan^{-1} \left(\frac{\sqrt{x^3 + y^3}}{\sqrt{x} + \sqrt{y}} \right),$$

then show that : [7]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -2 \sin^3 u \cos u.$$

(c) If

$$u = x^2 - y^2, \quad v = 2xy \quad \text{and} \quad z = f(u, v),$$

then show that :

[6]

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2} \frac{\partial z}{\partial u}.$$

7. (a) If

[4]

$$u + v = x^2 + y^2, \quad u - v = x + 2y$$

Find $\frac{\partial u}{\partial x}$ treating y constant.

(b) Examine for functional dependence :

[4]

$$u = \frac{x-y}{x+z}, \quad v = \frac{x+z}{y+z}.$$

(c) Find stationary points of :

[5]

$$f(x, y) = x^3 y^2 (1 - x - y)$$

and find f_{\max} where it exists.

Or

8. (a) If

[4]

$$x = v^2 + w^2, \quad y = w^2 + u^2, \quad z = u^2 + v^2,$$

prove that $JJ' = 1$.

109

- (b) Find the percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula : [4]

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

if r_1, r_2, r_3 are in error by 2% each.

- (c) Find the stationary points of :

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

if the condition $4x^2 + y^2 + 4z^2 = 16$ is satisfied. [5]